Globally Solving Nonconvex Quadratic Programming Problems
via Completely Positive Programming

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Contribution: Global Solver for Nonconvex QP

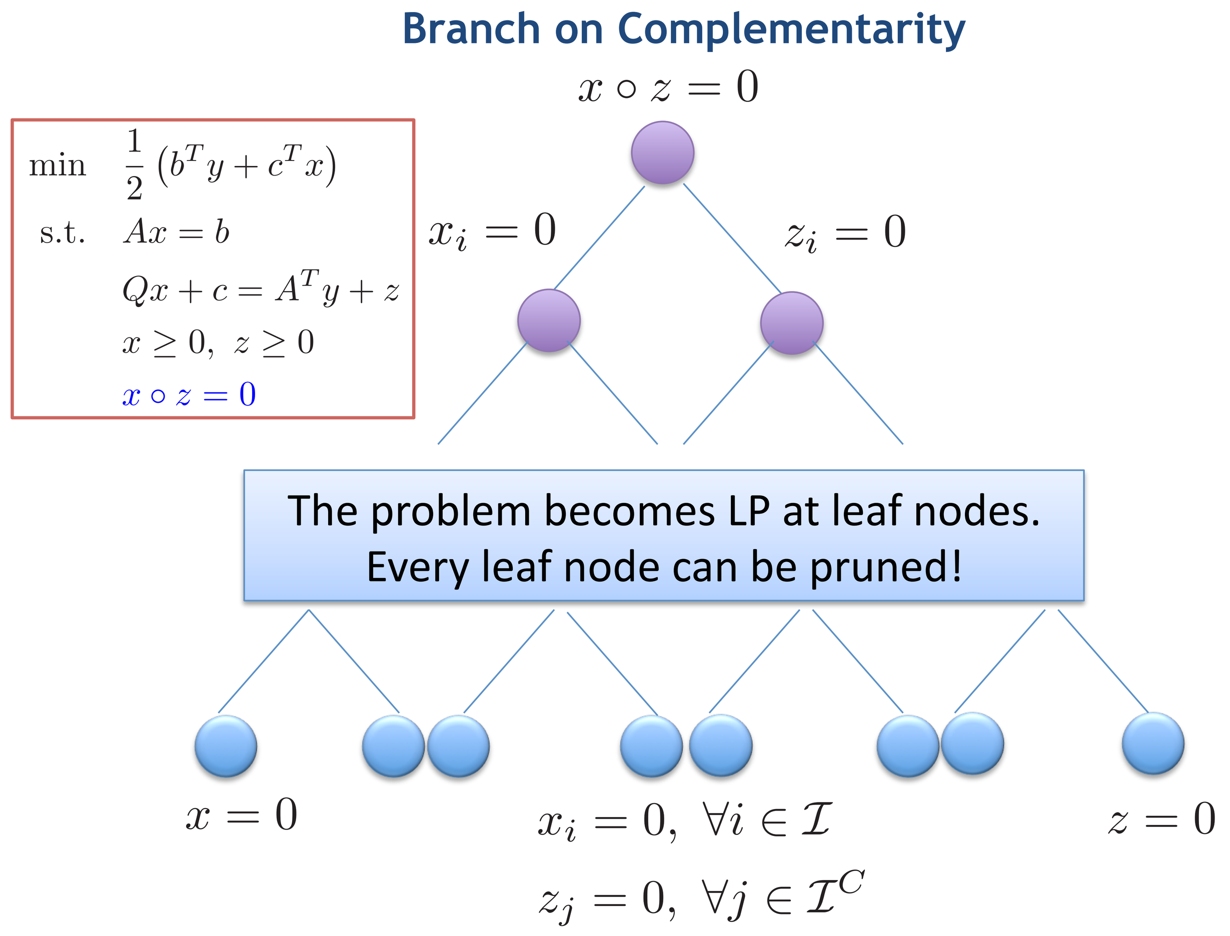
The combination of a finite branch-and-bound method and semidefinite relaxation, which is based on completely positive programming, results in a global optimization solver that is competetive with the state-of-the-art global solvers and more robust.

Nonconvex QP

$$\begin{aligned} \min \quad & \tfrac{1}{2}x^T H x + f^T x \qquad \text{(QP)} \\ \text{s.t.} \quad & Ax \le b \\ & A_{eq}\, x = b_{eq} \\ & l \le x \le u, \end{aligned}$$

- H is *not* positive semidefinite, and (QP) is nonconvex and NP-hard.
- Assumption: feasible region is bounded.
- Many applications in science and engineering including inventory theory, scheduling, robust linear regression and so on.
- Frequently appears as sub-structure in more complex optimization problems.

Finite B&B Method



The finite B&B method by Burer and Vandenbussche (2008) works by enforcing the compelementarity conditions through branching. Two major advantages of this approach:

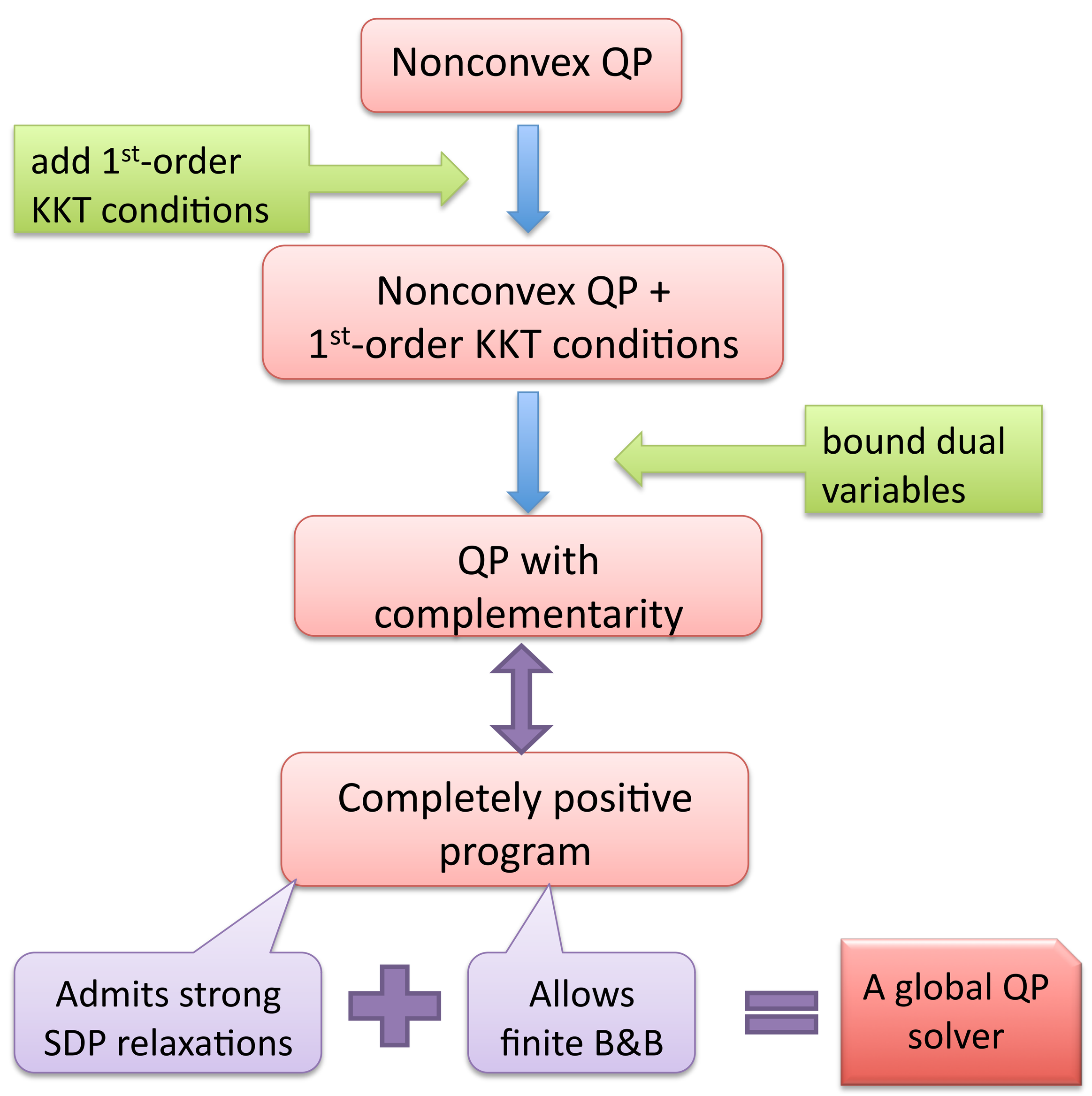
- The B&B tree is finite, unlike spatial branching (possibly infinite)
- Promotes stronger relaxations due to the incorporation of KKT conditions.

Semidefinite Relaxations

Strong semidefinite relaxations derived from completely positive programming:

$$\begin{aligned} \min \quad & \tfrac{1}{2}H \bullet X + f^T x \\ \text{s.t.} \quad & Ax = b \\ & \operatorname{diag}(AXA^T) = b \circ b \\ & X_{ij} = 0 \ \ \forall\, (i,j) \in E \\ & \begin{pmatrix} 1 & x^T \\ x & X \end{pmatrix} \succeq 0 \\ & (x, X) \ge 0 \end{aligned}$$

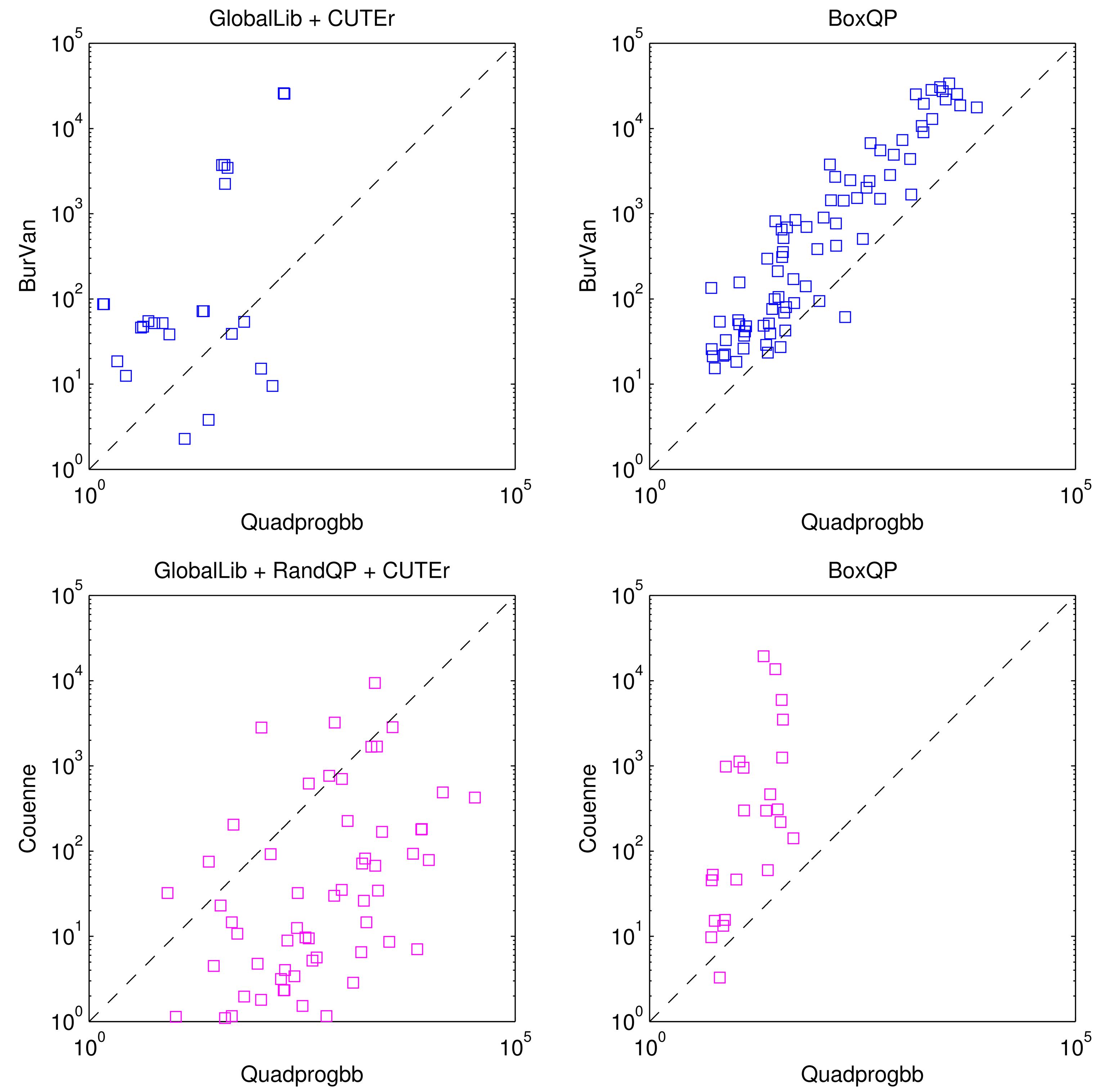
Reformulation



Experiment Data

Type	# Instances	# Vars	# Cons	*H* Density
BoxQP	90	[20, 100]	[0, 0]	[0.19, 0.99]
Globallib	83	[2, 100]	[1, 52]	[0.01, 1]
RandQP	64	[20, 50]	[14, 35]	[0.23, 1]
CUTEr	6	[4, 12]	[0, 13]	[0.08, 1]

Comparison with State-of-the-Art Solvers



Comparison with BurVan.

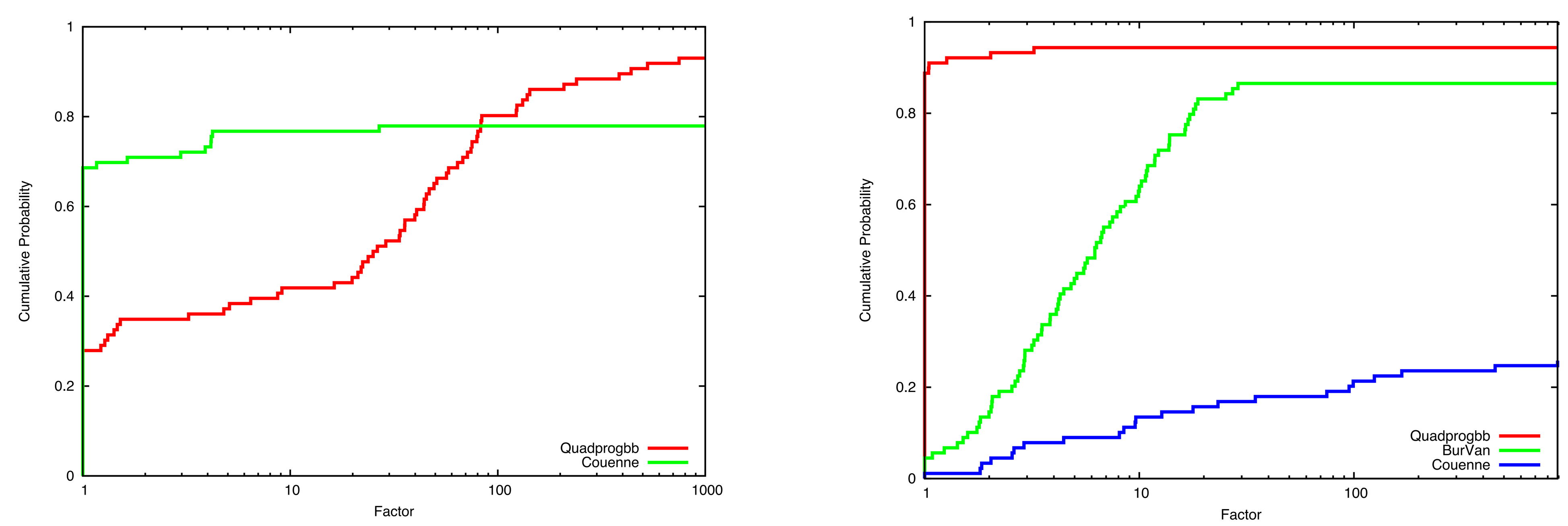
BurVan is an SDP based B&B code for nonconvex QPs. Each square's x- and y-cordinate represents times for our code and BurVan, repsectively. Clearly, our code if faster than BurVan.

Comparison with Couenne.

Couenne is a modern general-purpose global optimization solver. Our code is significantly faster than Couenne on BoxQP instances, but slower on other type of instances. However, our code is better in closing the gap on hard instances and uses much less nodes than Couenne.

Performance Profiles

For Globallib, RandQP and CUTEr instances, Couenne is faster than Quadprogbb on many of them but could not solve as many instances as Quadprogbb. On BoxQP instances, Quadprogbb is clearly the



winner among all the three methods: not only it is the fastest, but also it solved more hard instances. In summary, Quadprogbb is competitive in terms of speed and more robust.

Reference

[1] Jieqiu Chen, Samuel Burer, "Globally Solving Nonconvex Quadratic Programming Problems via Completely Positive Programming." *Math. Programming Computation*, to appear, 2011.

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Source code is available at: www.mcs.anl.gov/~jieqchen/.
The solver can be called in MATLAB and used in YALMIP.
Third party sofware: CPLEX