MINOTAUR: A new MINLP Solver

Next-Generation Solvers for Mixed-Integer Nonlinear Programs: Structure, Search, and Implementation

Sven Leyffer, *Ashutosh Mahajan*, and Todd S. Munson
Mathematics and Computer Science Division
Argonne National Laboratory
9700 S Cass Avenue, Argonne, IL 60439

Abstract

We present MINOTAUR, a next-generation open-source solver for mixed-integer nonlinear programming (MINLP). Minotaur has been built ground-up for solving MINLP and global optimization problems. It offers a variety of algorithms: Branch-and-Bound, Quesada-Grossmann, Branch-and-Reduce for solving these problems. Minotaur is more than a collection of solvers. It is a toolkit that provides several methods to enable researchers and users to implement new algorithms with minimal effort. The modular design of Minotaur makes it possible to create problem specific solvers by mixing and customizing MINLP techniques for creating relaxations, solving subproblems, branching, selecting nodes, heuristic search, presolving, bound tightening etc. In this talk, we will first describe briefly the design of Minotaur toolkit, and compare the performance of its default algorithms with that of existing state-of-the-art solvers. Next, we will describe the following new mathematical techniques that we have developed and implemented in Minotaur.

Exploiting Convexity. We have implemented a new technique to check if the feasible region of certain nonlinear functions can be represented as a union of few convex regions. By branching in a specific way, we can create convex subproblems of these nonlinear problems. Such a branching scheme can speed up by orders of magnitude, the global optimization of problems with specific structure. We describe the necessary and sufficient conditions for this property, and show computational results.

QP Diving Based Search. Quadratic programming (QP) approximations of a nonlinear problem can be solved much faster than the nonlinear problem. More importantly for branch-and-bound based methods, QP approximations capture second-order information of the nonlinear constraints and objectives, something which LP relaxations do not, and still have good warm-starting capability. We exploit this property to develop a QP based diving algorithm for branch-and-bound. A QP approximation of the problem is solved at a node. We branch on a fractional variable, and continue to re-solve this QP at subsequent nodes until certain conditions are satisfied. We compare using computational results on benchmark instances, different strategies of diving and creating approximations.

Presolve. Most MINLP solvers only use presolving methods to simplify linear relaxations of MINLP. For these methods, they rely on techniques developed for mixed-integer linear programming. Minotaur, on the other hand, offers methods and data-structures to analyze and manipulate nonlinear functions directly. Taking advantage of these methods, we deploy several presolving methods to simplify MINLP instances. By working in the space of original variables and constraints, we are able to reduce problems more effectively and create tighter relaxations. We will describe these methods and demonstrate their effect on benchmark instances.