Scalable implicit algorithms for stiff hyperbolic PDE systems

Enabling Predictive Extended MHD Simulations by the Development of Stable, Accurate and Scalable Computational Formulations and Solution Methods

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Abstract

The need to solve systems of hyperbolic PDEs is ubiquitous and pervasive within the DOE complex. Well-known mathematical models such as Euler, Navier-Stokes, and MHD (all of wide applicability in DOE missions) are all fundamentally hyperbolic. Such systems typically support fast dynamics (e.g, advection, wave propagation). Often, the modeler is interested in following such fast dynamics, as they are relevant for understanding important phenomena such as shock propagation, turbulence dynamics, etc. However, there are situations where fast time-scales may interact and balance each other to produce much slower dynamics or nontrivial steady-state solutions. It is the latter scenario which is of interest in the context of this work.

Algorithmically, much emphasis has been placed in the development of suitable spatio-temoral discretization techniques for hyperbolic PDEs. In the context of fast hyperbolic phenomena, a major research focus has been the development of accurate shock-capturing spatial discretization techniques. In this case, the temporal integration scheme has typically remained explicit (i.e., one in which only old-time solution information is employed to find the new-time state). Explicit schemes are by design conceptually simple, but suffer from numerical stability time-step constraints determined by the fastest dynamics supported by the hyperbolic model. While such constraints are acceptable for the modeling of fast hyperbolic phenomena, they typically demand onerous computing requirements when slower dynamics are of interest.

Implicit time-stepping bypasses such numerical time-step stability constraints, and thus offers a *potentially* efficient algorithmic avenue to follow slow dynamics or to compute steady-state configurations. Implicit techniques employ both old- and new-time solution information to update the new-time state. As a result, new-time solution information couples with itself in a non-trivial manner, giving way to a large set of (nonlinear) algebraic equations that needs to be solved to find the new-time solution. Thus, the actual algorithmic performance of implicit time-stepping methods ultimately depends on the availability of robust, scalable, and efficient nonlinear solvers.

However, the development of such implicit solvers for hyperbolic PDEs has remained challenging, owing to the ill-conditioning of the associated algebraic systems induced by the time-scale disparity. Moreover, such algebraic systems are neither symmetric nor diagonally dominant, which hamper the application of modern optimal iterative methods such as multilevel techniques. In this talk, we will describe the current state of the art in the formulation and development of viable nonlinear solver strategies for various stiff nonlinear hyperbolic PDE systems, including incompressible Navier-Stokes, resistive MHD, and extended MHD. The focus here is on Newton-Krylov nonlinear methods, which employ a Newton-Raphson iterative approach to find the nonlinear solution. A crucial step in the algorithm is the inversion of the linearized (Jacobian) algebraic system, which is performed by the Krylov method. The effectiveness of Krylov iterative techniques depends strongly on suitable preconditioning. In this talk, we will describe recent developments in multilevel-friendly block-preconditioning strategies for stiff hyperbolic systems, which enable scalable solvers both from the algorithmic and the parallel performance standpoints.