A New Lagrangian-Eulerian Approach to Oldroyd-B Viscoelastic Flow

Modeling and Algorithmic Approaches to Constitutively-Complex, Micro-structured Fluids

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Abstract

Viscoelastic flow of an Oldroyd-B fluid is described constitutively by coupling the upper-convected Maxwell PDE for the polymer stress to the incompressible Navier-Stokes through an extra forcing in the momentum equation. Simulation of benchmark flows for this class of problems in otherwise benign geometries, such as contraction channels and confined cylinders, has perplexed the non-Newtonian fluid mechanics community for years. Some believe that the mathematical system is ill-posed, but this has not been shown conclusively. Others believe that numerical methods are deficient, unable to capture fine spatial and temporal scales admitted by the mathematically mixed type system.

In this work we acknowledge the former model complaint but focus on the latter numerical problem. We will show that stable and convergent solutions to this system are possible with high resolution numerical methods. Our approach has been the use of adaptive mesh refinement to gain significant resolution in the problematic areas of the domain. However, due to differing temporal scales between the fluid velocity and the polymer stress, the equilibration of the solution requires excessively long times. Our new approach is based on a new hybrid Lagrangian-Eulerian scheme that solves for the steady-state velocity and pressure using the aforementioned adaptive grid based method, while the stress is obtained by integrating the steady-state Oldroyd-B ODE along streamlines. This approach allows for higher resolution of the stress in problematic areas, notably in the vicinity of stagnation points, and at significantly lower computational cost. We believe that by using this approach we will be able to achieve stable steady-state solutions for values of the Weissenberg number that exceed the current reliable benchmarks. Another added value of our approach is a better understanding of the small scale structures that have hindered so far the success of standard PDE-based methods.