A Polynomial Time Constraint Reduced Algorithm for Semidefinite Optimization Problems

Interior-Point Algorithms for Optimization Problems with Many Constraints
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Abstract

We propose a new infeasible primal-dual predictor-corrector interior-point method (IPM) with adaptive criteria for constraint reduction. The algorithm is a modification of one with no constraint reduction, due to Potra and Sheng (2006). Our algorithm can be applied when the data matrices are block diagonal. It thus solves as special cases any optimization problem that is linear, convex quadratic, convex quadratically constrained, or a second-order cone problem.

We verify the validity of the algorithm by proving global convergence. We also prove its polynomial complexity, $O(n \ln(\epsilon_0/\epsilon))$ iterations, for a given convergence tolerance ϵ and an initial residual ϵ_0. This is the first such complexity result for primal-dual constraint-reduction algorithms for any class of problems.

The work is documented in Park (2011) and continues DOE-supported work from our research group cited in the references below.

The importance of constraint-reduction algorithms is in solving problems with very many inequality constraints. Such problems arise, for example, in the training of support vector machines, in optimization problems with partial differential equation constraints, and in relaxations of integer programming problems such as the maximum binary code problem, the traveling salesperson problem, and the quadratic assignment problem.

References:

Jin Hyuk Jung, Dianne P. O'Leary, and André L. Tits. Adaptive constraint reduction for training support vector machines. *Electronic Transactions on Numerical Analysis*, 31:156–177, 2008.

Jin Hyuk Jung, Dianne P. O'Leary, and André L. Tits. Adaptive constraint reduction for convex quadratic programming. *Computational Optimization and Applications*, 2010. DOI:10.1007/s10589-010-9324-8.

Sungwoo Park. *Matrix Reduction in Numerical Optimization.* PhD thesis, Computer Science Department, University of Maryland, College Park, MD, 2011.
http://drum.lib.umd.edu/handle/1903/11751

Florian A. Potra and Rongqin Sheng. A superlinearly convergent primal-dual infeasible-interior-point algorithm for semidefinite programming. *SIAM Journal on Optimization*, 8(4):1007–1028, 2006.

A. L. Tits, P. A. Absil, and W. Woessner. Constraint reduction for linear programs with many constraints. *SIAM Journal on Optimization*, 17(1):119–146, 2006.

Luke B. Winternitz, Stacey O. Nicholls, André L. Tits, and Dianne P. O'Leary. A constraint-reduced variant of Mehrotra's predictor-corrector algorithm. *Computational Optimization and Applications*, 2011.