Agglomeration-based $H(\mathrm{curl})$ and $H(\mathrm{div})$ coarse spaces with approximation properties

Constrained Energy Minimization Discretization and Multigrid Methods

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Abstract

We extend previously studied technique based on element agglomeration for constructing coarse subspaces of the lowest order Nédélec and Raviart–Thomas finite element spaces on general unstructured tetrahedral meshes. Provided that agglomerated elements, viewed as polyhedral domains, possess certain regular topology (such as properly defined faces and edges), the constructed coarse spaces form a de Rham complex, like the fine–grid ones. In contrast to the previous work, the new coarse spaces preserve locally given set of functions and thus can ensure certain order of approximation. For example, the coarse spaces can be constructed to interpolate vector constants locally on each agglomerate, which allows to approximate locally smooth functions with first order. On the other hand, for the PDEs with high-contrast coefficients, different choices of functions to be preserved locally can lead to efficient operator-dependent interpolation needed in two- and multilevel solvers.

The introduced coarse spaces have two main applications - numerical upscaling and multilevel solvers. We demonstrate the improved performance of the operator-dependent coarse Raviart-Thomas spaces using the SPE10 data set. Numerical results illustrating the properties of the $H(\mathbf{curl})$-coarse spaces are presented as well.