GPU Accelerated High-Order Hybrid Methods for Shock Problems

Parallel High-Order Accuracy Methods Applied to Non-Linear Hyperbolic Equations
and to Problems in Material Science

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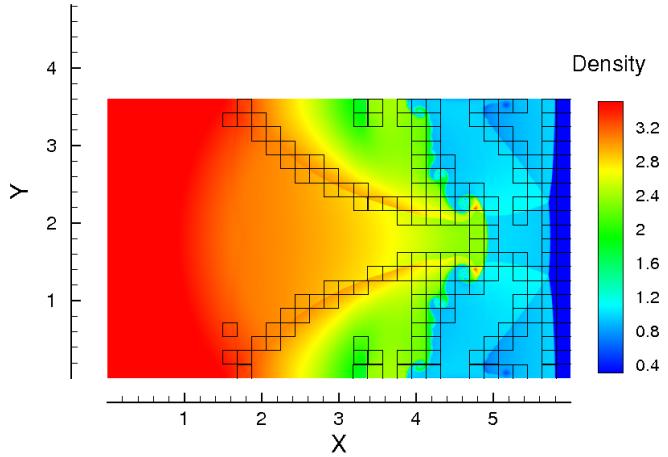
Abstract

While the importance of using high-order accurate computational methods for solving time-dependent partial differential equations with smooth is well accepted, the situation for problems with shocks and unresolved steep gradients is less clear. On the other hand, for large-scale problems, the use of non-oscillatory schemes such as ENO/WENO schemes often leads to very expensive computational models in particular for multiscale and multiphysics problems.

In this talk we shall discuss the development of high-order hybrid schemes, combining the use of WENO methods where the solution lacks smoothness with spectral methods in regions where the solution is sufficiently smooth. The switching between the two types of approximation is done fully dynamically and controlled by a multiresolution analysis. While we shall briefly discuss the use of polynomial spectral methods, we focus the discussion of a recently developed Fourier continuation method that enables the use of Fourier spectral methods on finite domains. This is highly beneficial for both overall efficiency, coupling between different regions, and the size of the stable time-step.

We discuss the different elements of this scheme and show a number of examples to validate the accuracy and benefits of both the hybrid approach and the specific scheme based on Fourier methods. We shall also demonstrate the ability to use this approach on complex problems such as the Ritchmyer-Meshkov instability.

Time permitting we conclude by discussing the implementation of the dynamic hybrid scheme on GPGPU's as an example of what kind of performance one can expect for complex algorithms such as the one introduced here.



Initial stage of Ritchmyer-Meshkov instability of Xenon-Argon interface excited by a Mach 4.46 shock. Blocks indicate regions where a WENO scheme is applied while a lack of blocks indicate regions where a Fourier Continuation method is applied

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