Response-Excitation Theory for Stochastic ODEs and Stochastic PDEs (DOE grant number DE-FG02-07ER25818)

PI: George Em Karniadakis; Joint Work with Daniele Venturi

Division of Applied Mathematics, Brown University (RI, USA)

Abstract

By using functional integral methods we obtain new evolution equations for the *joint response-excitation* probability density function of arbitrary stochastic dynamical systems and stochastic PDEs subject to random initial conditions, random boundary conditions or random forcing terms. The proposed new framework allows us to deal effectively with colored noise sources and it seems more appropriate to tackle several open problems arising in the stochastic modeling of dynamical systems and PDEs such as long term integration (random frequency problem), curse of dimensionality, and discontinuities in random space, not easily handled with existing approaches. The new equations are always linear, even for nonlinear problems. In some cases, remarkably for *first-order stochastic PDEs* and for *stochastic dynamical systems subject to random initial conditions*, such evolution equation turns out to be a standard PDE which is very similar to the classical Fokker-Planck or the Liouville equation. We have obtained numerical solutions of these new equations involving the response-excitation PDF for specific model problems and compared the results against those obtained from stochastic collocation. We have found that the response-excitation approach indeed yields accurate representations of the statistical properties of the system. We will also discuss how to extend this theory to high-order stochastic PDEs.

The response-excitation theory is a fully consistent new approach to stochastic ODEs and stochastic PDEs that can potentially open new strands of reseach in different areas of mathematical physics, e.g., in the stochastic modeling of chemical, geological and biological systems, structural mechanics as well fluid and plasma dynamics.