Eigenanalysis for Galerkin Reformulations of Uncertain ODE Systems

Analysis and Reduction of Complex Networks Under Uncertainty

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Abstract

The integral curve of a system of ODEs with uncertain parameters will likewise be uncertain. Such a system can be reformulated into a system of Galerkin-projected ODEs that govern the coefficients of a (truncated) polynomial chaos (PC) representation of the (random) state variables. While this reformulated system is deterministic, it is of higher dimension. In this work we provide a framework for understanding the eigenstructure of the coefficient system and in what sense it represents the stochastic eigenstructure of the original uncertain system.

We will discuss how the location of the eigenvalues of the Jacobian of the projected system are largely determined by the locus of eigenvalues of the Jacobian of the original uncertain system, regardless of PC order or choice of orthogonal polynomials. In particular, the eigenvalues of the coefficient system Jacobian are always in the convex hull of the essential numerical range of the uncertain Jacobian [1]. Furthermore, as the order of representation increases, there is a weak convergence of the eigenvalues of the coefficient system Jacobian toward the locus of the eigenvalues of the uncertain Jacobian. This convergence is established using orthogonal matrix polynomials [2].

The Jacobian of the coefficient system has a large number of high-dimensional eigenvectors. We will discuss the relationship between these eigenvectors and the stochastic eigenvectors of the uncertain Jacobian [3].

This approach is demonstrated in the analysis of a system describing the concentration levels in the oxidation of CO on a surface (Makeev et al., JCP, 2002). This system is oscillatory, exhibits a slow manifold, and features a Hopf bifurcation for certain parameter values. These parameters are considered to be uniform random variables (over a domain of interest). The uncertain state of the dynamical system at a specified time is represented on a basis of orthogonal polynomials using non-intrusive spectral projection. The eigenstructure of the uncertain Jacobian at this uncertain state is compared with the eigenstructure of the corresponding PC coefficient system.

