Parameter Estimation Via Stochastic Non-Linear Programming

Project: Risk Management and Planning in Complex Networks

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Abstract

Nonlinear programming continues to be an effective tool for realizing efficiency improvements and cost reductions through optimization of designs and operations of various processes. The size of these optimization problems continues to grow as more rigorous models are developed and higher level operating concerns such as planning, scheduling, and uncertainty are considered. Rigorous models are desirable to reduce model error, the inclusion of planning and scheduling components allows optimizing over a longer time horizon than would be possible otherwise, and the inclusion of uncertainty yields more robust solutions. All of these considerations lead to increasingly large and complicated problems that can quickly outstrip the computing capabilities of current desktop computing using serial solution approaches. This is particularly true for the case of stochastic, non-linear optimization problems, whose computational difficulty is well-known.

In this work, we consider the efficient solution of a broad class of stochastic non-linear programming problem, that of model parameter estimation. We exploit a fundamental, unexplored equivalency between stochastic non-linear programming and parameter estimation problems. Specifically, parameter estimation problems can be expressed as non-linear stochastic programs, as they share multi-stage decision structure and the associated block-angular matrix form. The latter suggests that it may be possible to exploit advances in decomposition techniques for solving stochastic programs, in order to yield significant accelerations in the solution of parameter estimation problems. Further, parallelization of the decomposition strategy is likely to yield further, significant run-time reductions.

We use Sandia's Pyomo[1] package to model general non-linear parameter estimation problems. Decomposition strategies for stochastic programming, specifically Rockafellar and Wets' Progressive Hedging algorithm, are employed for problem solution. Specifically, we use Sandia's PySP [2] software library for stochastic programming. Ipopt [3] is used to solve individual sub-problems.

We demonstrate the potential of our approach for parameter estimation by computing transmission parameters in an SIR disease model using pre-vaccination measles data from 60 cities in England and Wales made available by Grenfell[4]. This estimation problem is formulated as a non-linear, multi-scenario problem with each city as a single scenario and the transmission parameters as common (first-stage) variables. The size of the cities are used to weight the likelihood of each scenario with larger cities receiving greater weights. The full paper describes in detail the problem formulation and estimation approach along with the resulting solutions. We discuss our experience in solving the disease parameter estimation problem on a parallel computing cluster, contrasting results to more direct solution methods.

[1]https://software.sandia.gov/trac/coopr/wiki/Pyomo
[2]https://software.sandia.gov/trac/coopr/wiki/PySP
[3]http://www.coin-or.org/Ipopt
[4]http://www.zoo.cam.ac.uk/zoostaff/grenfell/measles.htm