MOPEC: multiple optimization problems with equilibrium constraints

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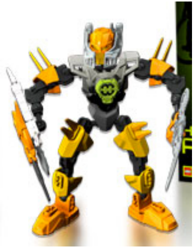
Building mathematical models

- How to model: pencil and paper, excel, Matlab, R, python, ...

  - Linear vs nonlinear
  - Deterministic vs probabilistic
  - Static vs dynamic (differential or difference equations)
  - Discrete vs continuous
- Other issues: large scale, tractability, data (rich and sparse)
- Abstract/simplify:
  - Variables: input/output, state, decision, exogenous, random...
  - Objective/constraints
  - Black box/white box
  - Subjective information, complexity, training, evaluation
- Must be able to model my problem easily/naturally
- Just solving a single problem isn't the real value of modeling: e.g. optimization finds "holes" in the model

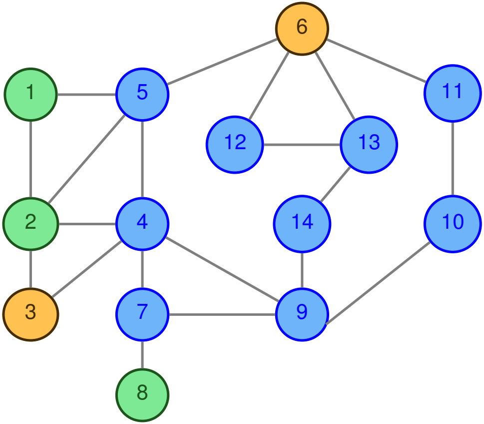
Why model?



- to understand (descriptive process, validate principles and/or explore underlying mechanisms)
- to predict (and/or discover new system features)
- to combine (engaging groups in a decision, make decisions, operate/control a system of interacting parts)
- to design (strategic planning, investigate new designs, can they be economical given price of raw materials, production process, etc)

Power System: Economic Dispatch

$$\min_{(q,z,\theta)\in\mathcal{F}} \sum_k C(q_k) \text{ s.t. } q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k$$



- Independent System Operator (ISO) determines who generates what
- p_k: Locational marginal price (LMP) at k
- Volatile in "stressed" system
- Can we shed load from consumers to smooth?
- FERC (regulator) writes the rules - how to implement?

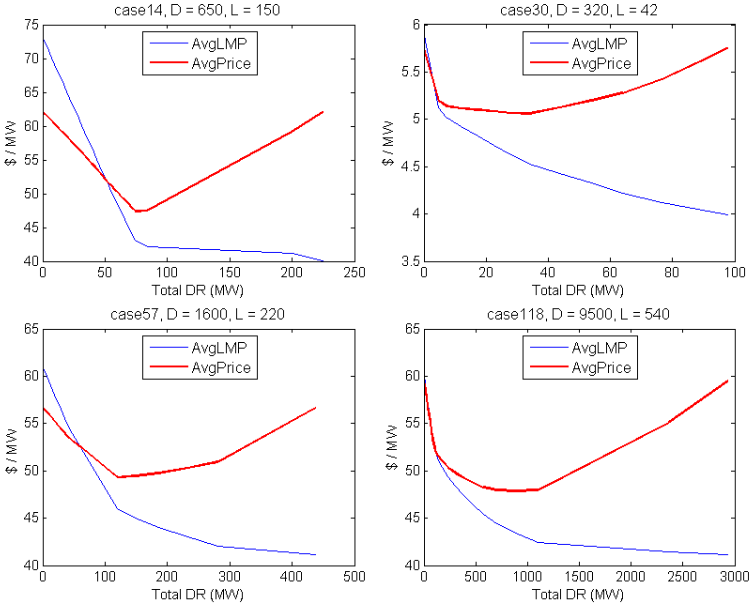
Understand: demand response and FERC Order No. 745

$$
\begin{aligned}
\min_{q,z,\theta,R,p} \; & \sum_k p_k R_k \\
\text{s.t.}\; & C_1 \geq \sum_k p_k d_k / \sum_k d_k \\
& C_2 \geq \sum_k (q_k + R_k) p_k / \sum_k (d_k - R_k) \\
& 0 \leq R_k \leq u_k,
\end{aligned}
$$

and (q, z, θ) solves

$$
\begin{aligned}
\min_{(q,z,\theta)\in\mathcal{F}} \quad & \sum_k C(q_k) \\
\text{s.t.} \quad & q_k - \sum_{(l,c)} z_{(k,l,c)} = d_k - R_k \qquad (1) \\
\text{where} \quad & p_k \text{ is the multiplier on constraint (1)}
\end{aligned}
$$

Stability and feasibility



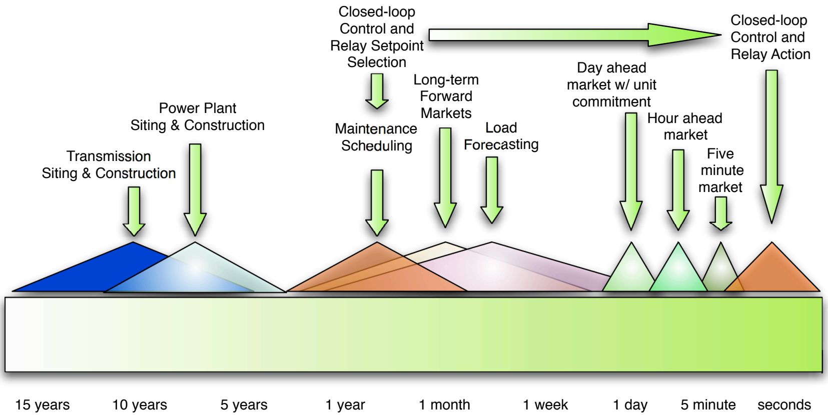
Insights

- Bilevel program (hierarchical model)
- Upper level objective involves multipliers on lower level constraint
- Extended Mathematical Programming (EMP) annotates model to facilitate communicating structure to solver
  - dualvar p balance
  - bilevel R min cost q z θ balance ...
- Automatic reformulation as an MPEC
- Model solved using NLPEC and Conopt
- Potential for solution of "consumer level" demand response
- Challenge: devise robust algorithms to exploit this structure for fast solution

Example: The smart grid

- The next generation electric grid will be more dynamic, flexible, constrained, and more complicated.
- Decision processes (in this environment) are predominantly hierarchical.
- Models to support such decision processes must also be layered or hierachical.
- Optimization and computation facilitate adaptivity, control, treatment of uncertainties and understanding of interaction effects.
- Developing interfaces and exploiting hierarchical structure using computationally tractable algorithms will provide FLEXIBILITY, overall solution speed, understanding of localized effects, and value for the coupling of the system.

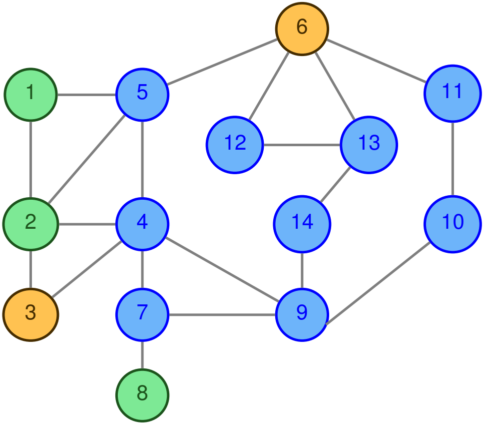
Representative decision-making timescales in electric power systems



A monster model is difficult to validate, inflexible, prone to errors.

Combine: Transmission Line Expansion Model

$$\min_{x \in X} \sum_{\omega} \pi_\omega \sum_{i \in N} d_i^\omega p_i^\omega(x)$$



- Nonlinear system to describe power flows over (large) network
- Multiple time scales
- Dynamics (bidding, failures, ramping, etc)
- Uncertainty (demand, weather, expansion, etc)
- $p_i^\omega(x)$: Price (LMP) at i in scenario ω as a function of x
- Use other models to construct approximation of $p_i^\omega(x)$

Generator Expansion (2): $\forall f \in F$:

$$\min_{y_f} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j)$$

$$\text{s.t.} \sum_{j \in G_f} y_j \leq h_f, y_f \geq 0$$

G_f: Generators of firm $f \in F$
y_j: Investment in generator j
q_j^ω: Power generated at bus j in scenario ω
C_j: Cost function for generator j
r: Interest rate

Market Clearing Model (3): $\forall \omega$:

$$\min_{z,\theta,q^\omega} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \text{s.t.}$$

$$q_j^\omega - d_j^\omega = \sum_{i \in I(j)} z_{ij} \quad \forall j \in N(\perp p_j^\omega)$$

$$z_{ij} = \Omega_{ij}(\theta_i - \theta_j) \quad \forall (i,j) \in A$$

$$-b_{ij}(x) \leq z_{ij} \leq b_{ij}(x) \quad \forall (i,j) \in A$$

$$\underline{u}_j(y_j) \leq q_j^\omega \leq \overline{u}_j(y_j)$$

z_{ij}: Real power flowing along line ij
q_j^ω: Real power generated at bus j in scenario ω
θ_i: Voltage phase angle at bus i
Ω_{ij}: Susceptance of line ij
$b_{ij}(x)$: Line capacity as a function of x
$\underline{u}_j(y)$, $\overline{u}_j(y)$: Generator j limits as a function of y

How to combine: Nash Equilibria

- Non-cooperative game: collection of players $a \in \mathcal{A}$ whose individual objectives depend not only on the selection of their own strategy $x_a \in C_a = \mathrm{dom} f_a(\cdot, x_{-a})$ but also on the strategies selected by the other players $x_{-a} = \{x_a : o \in \mathcal{A} \setminus \{a\}\}$.
- Nash Equilibrium Point:

$$\bar{x}_{\mathcal{A}} = (\bar{x}_a, a \in \mathcal{A}) : \forall a \in \mathcal{A} : \bar{x}_a \in \mathrm{argmin}_{x_a \in C_a} f_a(x_a, \bar{x}_{-a}).$$

1. for all $x \in \mathcal{A}$, $f_a(\cdot, x_{-a})$ is convex
2. $C = \prod_{a \in \mathcal{A}} C_a$ and for all $a \in \mathcal{A}$, C_a is closed convex.

VI reformulation

Define

$$G : \mathbb{R}^N \mapsto \mathbb{R}^N \text{ by } G_a(x_{\mathcal{A}}) = \partial_a f_a(x_a, x_{-a}), a \in \mathcal{A}$$

where ∂_a denotes the subgradient with respect to x_a. Generally, the mapping G is set-valued.

Theorem

Suppose the objectives satisfy (1) and (2), then every solution of the variational inequality

$$x_{\mathcal{A}} \in C \text{ such that } -G(x_{\mathcal{A}}) \in N_C(x_{\mathcal{A}})$$

is a Nash equilibrium point for the game.
Moreover, if C is compact and G is continuous, then the variational inequality has at least one solution that is then also a Nash equilibrium point.

Solution approach

- Use derivative free method for the upper level problem (1)
- Requires $p_i^\omega(x)$
- Construct these as multipliers on demand equation (per scenario) in an Economic Dispatch (market clearing) model
- But transmission line capacity expansion typically leads to generator expansion, which interacts directly with market clearing
- Interface blue and black models using Nash Equilibria (as EMP):

empinfo: equilibrium
forall f: min expcost(f) y(f) budget(f)
forall ω: min scencost(ω) q(ω) ...

Feasibility

$$\text{KKT of} \min_{y_f \in Y} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j) \qquad \forall f \in F \quad (2)$$

$$\text{KKT of} \min_{(z,\theta,q^\omega) \in Z(x,y)} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \qquad \forall \omega \quad (3)$$

- Models (2) and (3) form a complementarity problem (CP via EMP)
- Solve (3) as NLP using global solver (actual $C_j(y_j, q_j^\omega)$ are not convex), per scenario (SNLP) this provides starting point for CP
- Solve (KKT(2) + KKT(3)) using EMP and PATH, then repeat
- Identifies CP solution whose components solve the scenario NLP's (3) to global optimality

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

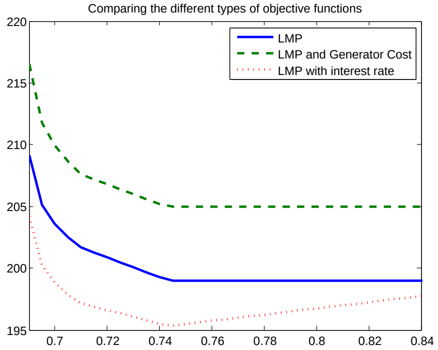
EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.34	4.62	5.01	3.99
ω_2		4.71	4.07	4.25	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	0.00	622.02			377.98
f_2			283.22	216.79	

Observations

- But this is simply one function evaluation for the outer "transmission capacity expansion" problem
- Number of critical arcs typically very small
- But in this case, p_j^ω are very volatile
- Outer problem is small scale, objectives are open to debate, possibly ill conditioned



- Economic dispatch should use AC power flow model
- Structure of market open to debate
- Types of "generator expansion" also subject to debate
- Suite of tools is very effective in such situations

Design: Stochastic competing agent models (with Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must “hedge” against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Each agent minimizes:

$$u_a = \sum_s \pi_s \left(\kappa - f(q_{a,s,*})\right)^2$$

Budget time 0: $\sum_i p_{0,i} q_{a,0,i} + \sum_j v_j y_{a,j} \leq \sum_i p_{0,i} e_{a,0,i}$

Budget time 1: $\sum_i p_{s,i} q_{a,s,i} \leq \sum_i p_{s,i} \sum_j D_{s,i,j} y_{a,j} + \sum_i p_{s,i} e_{a,s,i}$

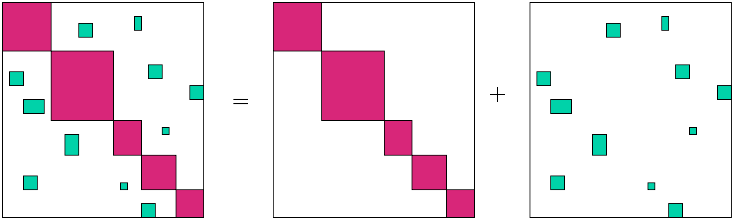
Additional constraints (complementarity) outside of control of agents:

$$\text{(contract) } 0 \leq -\sum_a y_{a,j} \perp v_j \geq 0$$

$$\text{(walras) } 0 \leq -\sum_a d_{a,s,i} \perp p_{s,i} \geq 0$$

Model and solve

- Can model financial instruments such as "financial transmission rights", "spot markets", "reactive power markets"
- Reduce effects of uncertainty, not simply quantify
- Use structure in preconditioners
  - Use nonsmooth Newton methods to formulate complementarity problem
  - Solve each "Newton" system using GMRES
  - Precondition using "individual optimization" with fixed externalities



Optimization models with explicit random variables

- Model transformation:
  - Write a core model as if the random variables are constants
  - Identify the random variables and decision variables and their staging
  - Specify the distributions of the random variables
- Solver configuration:
  - Specify the manner of sampling from the distributions
  - Determine which algorithm (and parameter settings) to use
- Output handling:
  - Optionally, list the variables for which we want a scenario-by-scenario report

Stochastic Programming as an EMP

Three separate pieces of information (extended mathematical program) needed

1. emp.info: model transformation

```
randvar F 2 discrete 0.25 0.8 // below
                     0.50 1.0 // avg
                     0.25 1.2 // above
cvarlo CVaR_r r alpha         // risk measure
stage 2 b s req               // multistage structure
```

2. solver.opt: solver configuration (benders, sampling strategy, etc)

```
4 "ISTRAT"  * solve universe problem (DECIS/Benders)
```

3. dictionary: output handling (where to put all the "scenario solutions")

Conclusions

- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Extended Mathematical Programming available within the GAMS modeling system
- Able to pass additional (structure) information to solvers
- Embedded optimization models automatically reformulated for appropriate solution engine
- Exploit structure in solvers
- Extend application usage further