The Fundmental Constants of Nature from Lattice Gauge Theory Simulations

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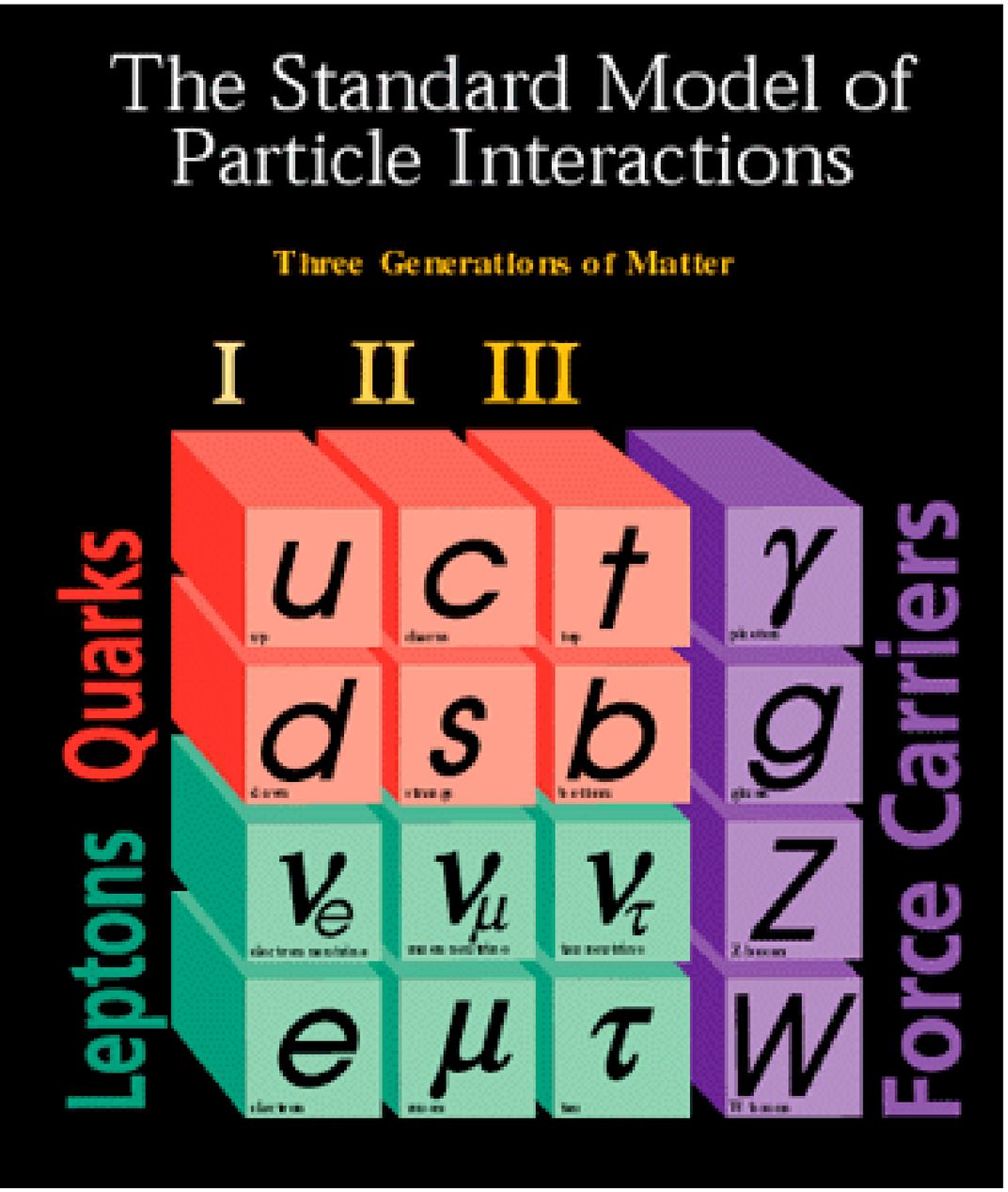
The Standard Model of particle physics



Describes the laws of nature at the most fundamental level now known.

- Six quarks and six leptons
  - (in three generations)
- 3 forces:
  - Strong interactions
  - Weak interactions
  - Electromagnetism

The parameters of the Standard Model

- Three coupling strengths:

$$\alpha_{\rm s},\ \alpha_{\rm w},\ \alpha_{\rm em}$$

- Six quark and six lepton masses

$$m_u,\ m_d,\ m_c,\ m_s,\ m_t,\ m_b$$
$$m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3}$$

- Mixings among the quarks (the Cabibbo-Kobayashi-Maskawa matrix, and (as of the last few years) among the leptons:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad \begin{pmatrix} V_{e\nu_1} & V_{e\nu_2} & V_{e\nu_3} \\ V_{\mu\nu_1} & V_{\mu\nu_2} & V_{\mu\nu_3} \\ V_{\tau\nu_1} & V_{\tau\nu_2} & V_{\tau\nu_3} \end{pmatrix}$$

Where do these parameters come from?
Can we predict them with a more fundamental theory?

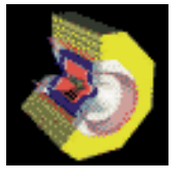
The Standard Model is *maddeningly* successful. It accounts for every particle physics experiment performed so far, sometimes to great precision (one part in a billion for the electron anomalous magnetic moment).

Why maddeningly?
It contains obvious gaps and puzzles.

- What is the Higgs?
- How can gravity be incorporated?
- Why is there more than one generation of quark?
- What is the relation between the three forces?
- ...

To to understand what physics lies "Beyond the Standard Model",

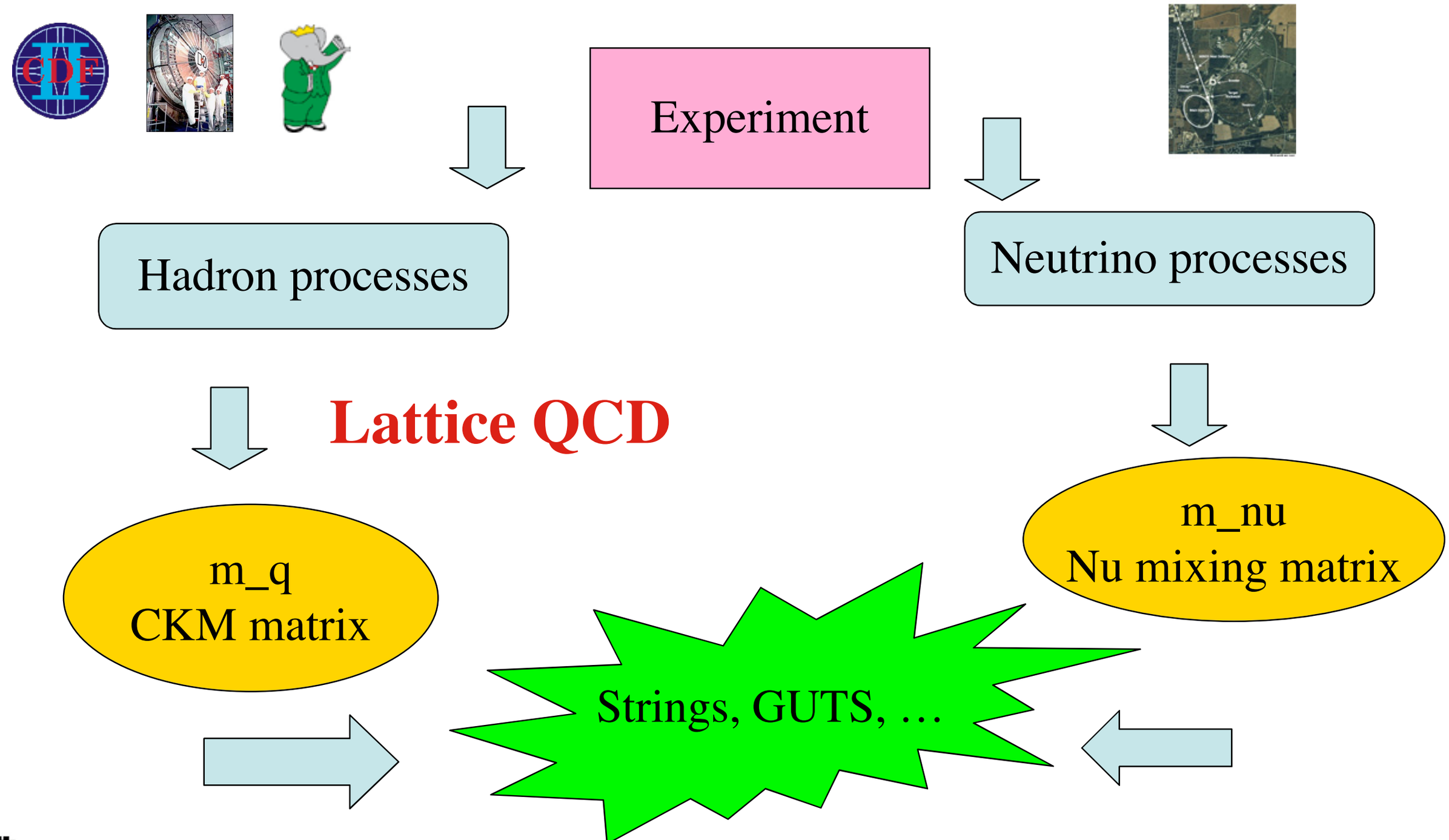
- Pin down the parameters of the Standard Model, so that they can be compared with Beyond the Standard Model theories.
  - Heavy flavor factories at SLAC, KEK, and Cornell are pouring out data to pin down the CKM matrix elements



- Search for new particles and forces.
  - Exhibit A: the Higgs. Proton colliders at Fermilab, and soon at the LHC at CERN are extending the search for the Higgs to higher and higher masses.

Lattice calculations are essential to this program in two ways.

First, they are required to extract quark properties from hadron properties.



Second, they are essential to prepare for possible new nonperturbative phenomena in coming experiments.

- LQCD is the first general tool for nonperturbative quantum field theory.
- Of the interactions known to particle physics, only one (quantum electrodynamics) is known to be described by a perturbative theory.
  - Consider the “Higgs”. Is it
    - Elementary?
    - A bound state of new strong interactions (“technicolor”, “topcolor”)?
    - Accompanied by very high energy gluino condensates (some supersymmetry modes)?

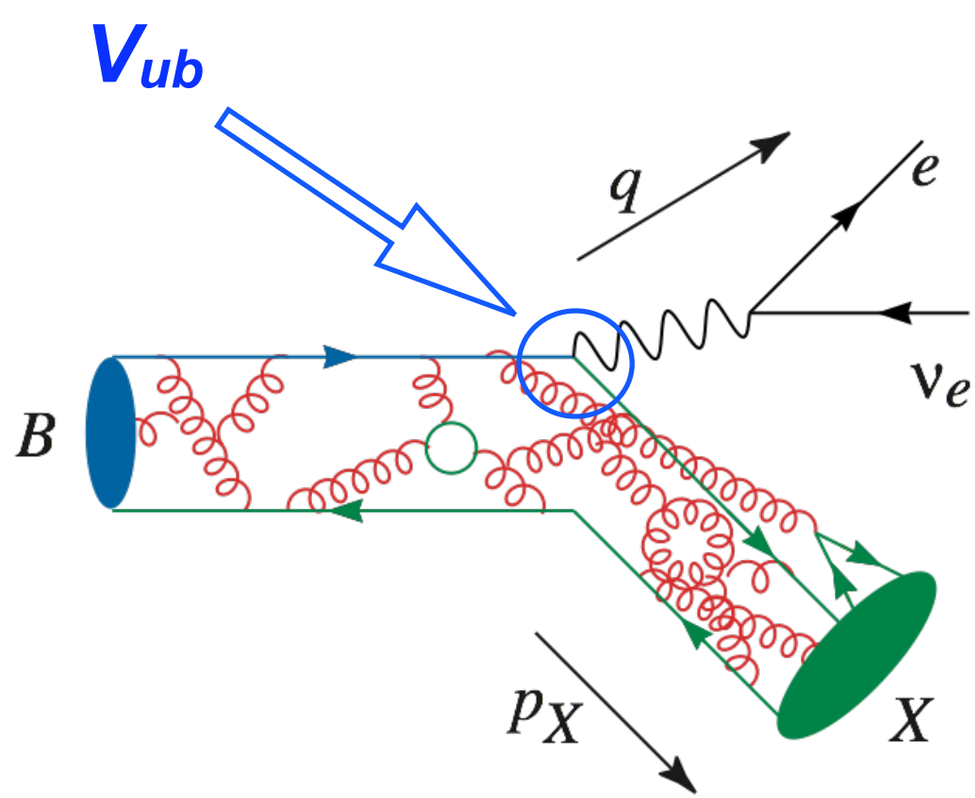
QCD provides an excellent test bed to sharpen our tools to prepare for such questions.

Quantum Chromodynamics (QCD)

Quark masses and mixings can't be directly observed.

Quarks are permanently confined inside hadrons, even though they behave as almost free particles as asymptotically high energies.
"Asymptotic freedom", Gross, Politzer, and Wilczek, Nobel Prize, 2004.

Quark properties must be inferred from observable hadron properties with lattice QCD calculations.



$B \to \pi l \nu$
semileptonic decay

Luke

Quantum field theories are defined by their path integrals.

$$Z = \int d[A_{x\mu}, \psi_x, \overline{\psi}_x] \exp\left(-S(A, \psi, \overline{\psi})\right)$$

Independent fields are defined at each point of space-time.

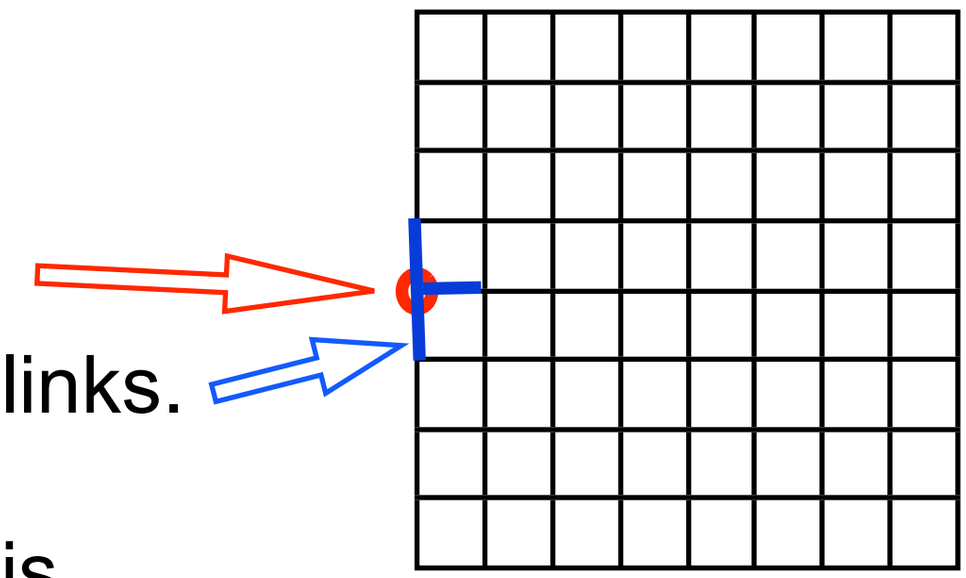
A continuum quantum field theory is in principle defined by an infinite dimensional integral (not a well-defined object).

QFTs must be “regulated”.

Lattice quantum field theories

Approximate the path integral by defining the fields on a four dimensional space-time lattice.

Quarks are defined on the sites of the lattice, and gluons on the links.



Continuum quantum field theory is obtained in the zero lattice spacing limit. This limit is computationally very expensive.

Operationally, lattice QCD computations consist of

1) Sampling a representative set of gauge configurations with Monte Carlo methods,

E.g., the Metropolis method, the hybrid Monte Carlo algorithm, ...

2) Calculating the propagation of quarks through the gauge configurations, and

Solve the Dirac equation on each configuration with relaxation methods,
e.g., biconjugate gradient algorithm, etc.

3) Constructing hadron amplitudes from the quark propagators.

Lattice spacing (fm)	Quark mass	Volume	Number of configurations
.15	.03	16^3x48	500
	.02		500
	.01		500
	...		...
.10	.03	28^3x96	500
	.02		500
	.01		500
	...		...
.06	.02	48^3x144	500
...	...	...	...

Creating 500 improved staggered configurations on 48^3x144 site lattices will take 0.5-1.5 TF-years, depending on the quark mass.

Current computing hardware state of the art: $1.3/MF.

Larger projects are ~ few TF-years.

This year's step in the project described in this talk:

Few TB of data

Generate new gauge configurations on a QCDOC at BNL.
2 TF-years.

A single highly optimized program,
very long single tasks,
moderate I/O.

Transfer to Fermlab for analysis on clusters.
0.7 TF-years.

Large, heterogeneous analysis code base,
small individual tasks,
heavy I/O.

Progress has come both from larger computers and improvement of methods.

Improved discretizations:

Numerical Analysis $\Rightarrow$

$$\frac{\partial \psi(x_j)}{\partial x} = \Delta_x \psi(x_j) + \mathcal{O}(a^2)$$

$$\frac{\psi(x_j + a) - \psi(x_j - a)}{2a}$$

$$\Rightarrow \quad \frac{\partial \psi}{\partial x} = \Delta_x \psi - \frac{a^2}{6} \Delta_x^3 \psi + \mathcal{O}(a^4)$$

10–15% for $a = 0.4\,\mathrm{fm}$

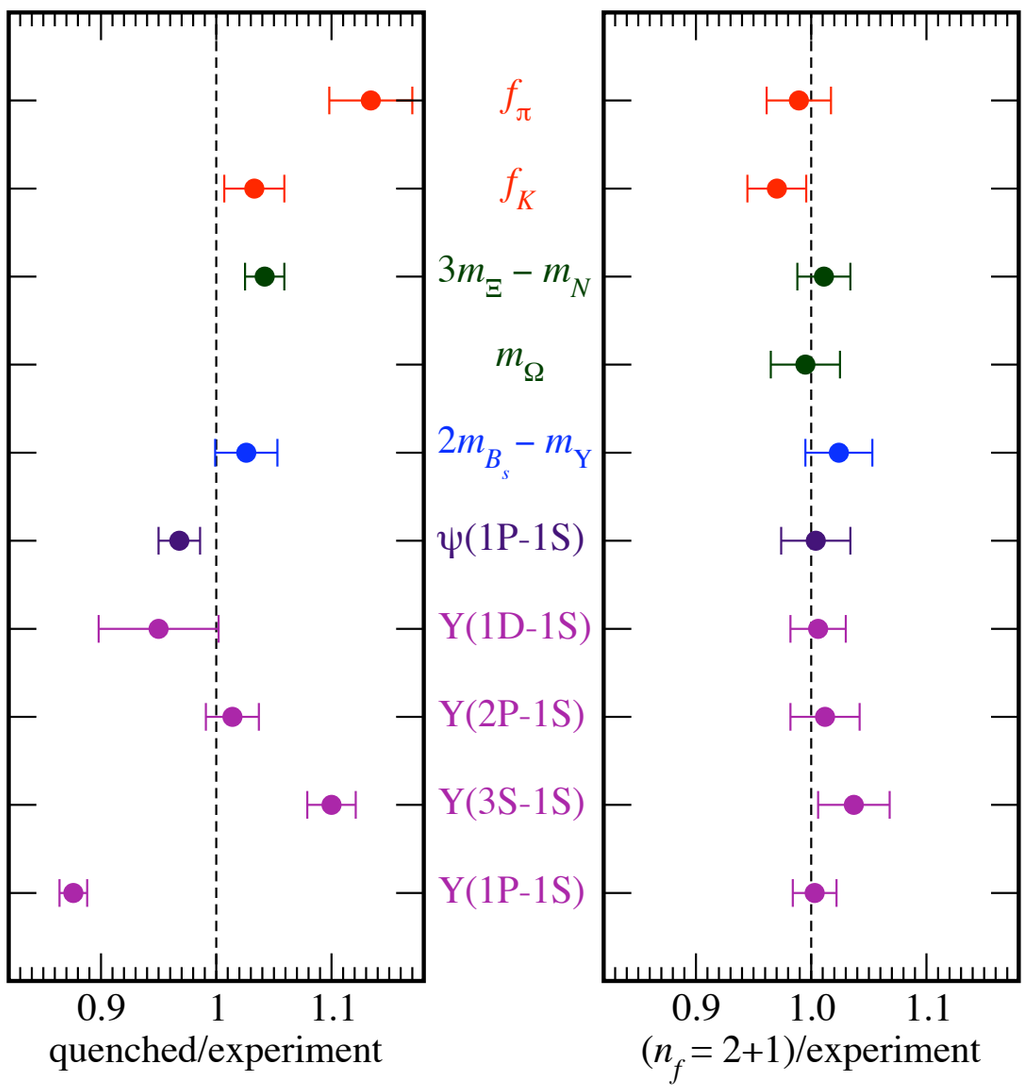
1–2% for $a = 0.4\,\mathrm{fm}$

$\Longrightarrow$ Improved staggered ("asqtad") fermions;

Naik; Orginos, Sugar, and Toussaint; Lepage.

Lattice QCD confronts experiment

Recent progress:



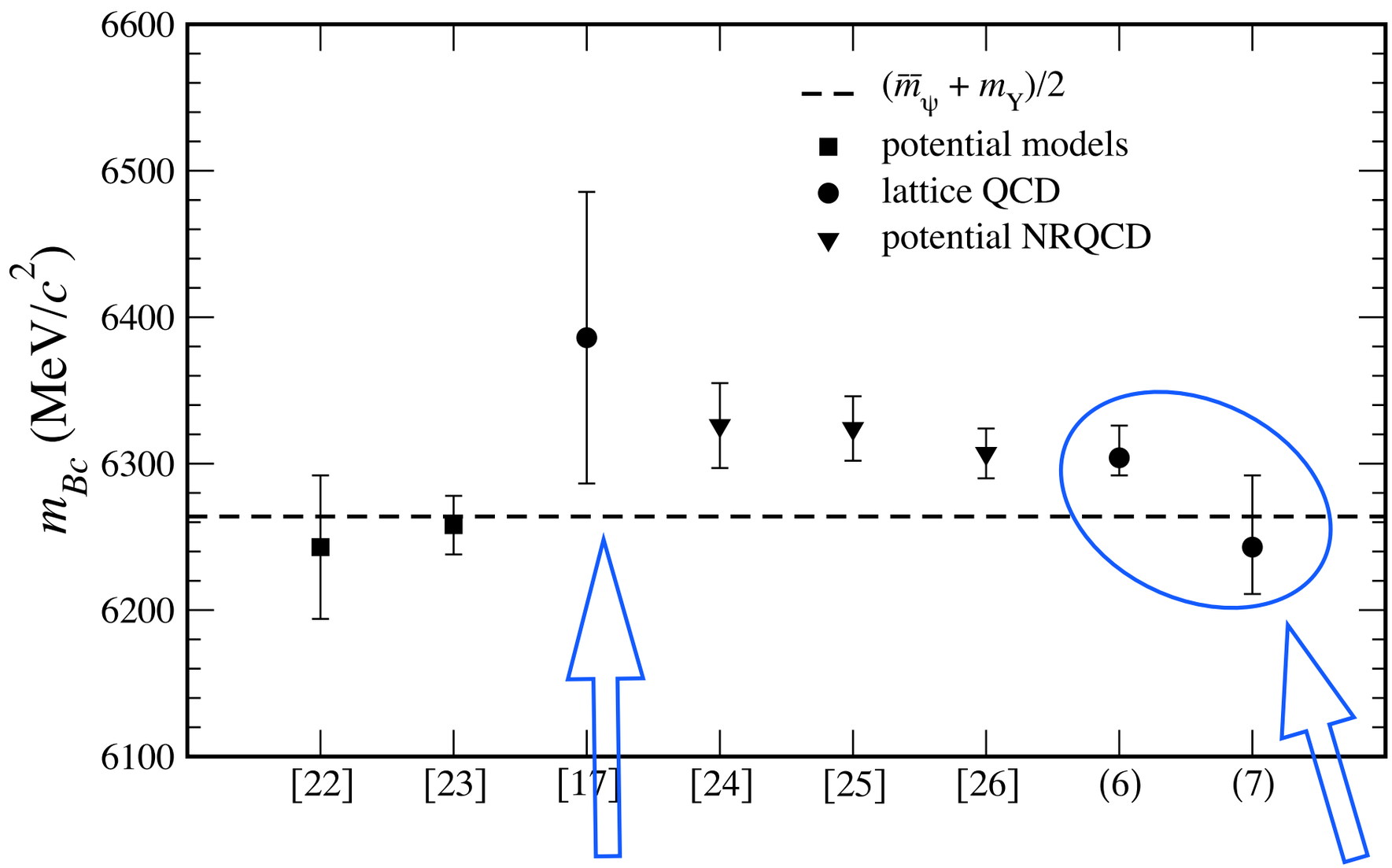
Davies *et al*, hep-lat/0304004 + Toussaint,Davies, LAT04

For simple quantities, the 10%-ish errors visible in the "quenched approximation" are removed with unquenched calculations using improved staggered fermions (the least computationally demanding fermion method).

Fermilab, HPQCD, MILC

A prediction: the mass of the B_c.

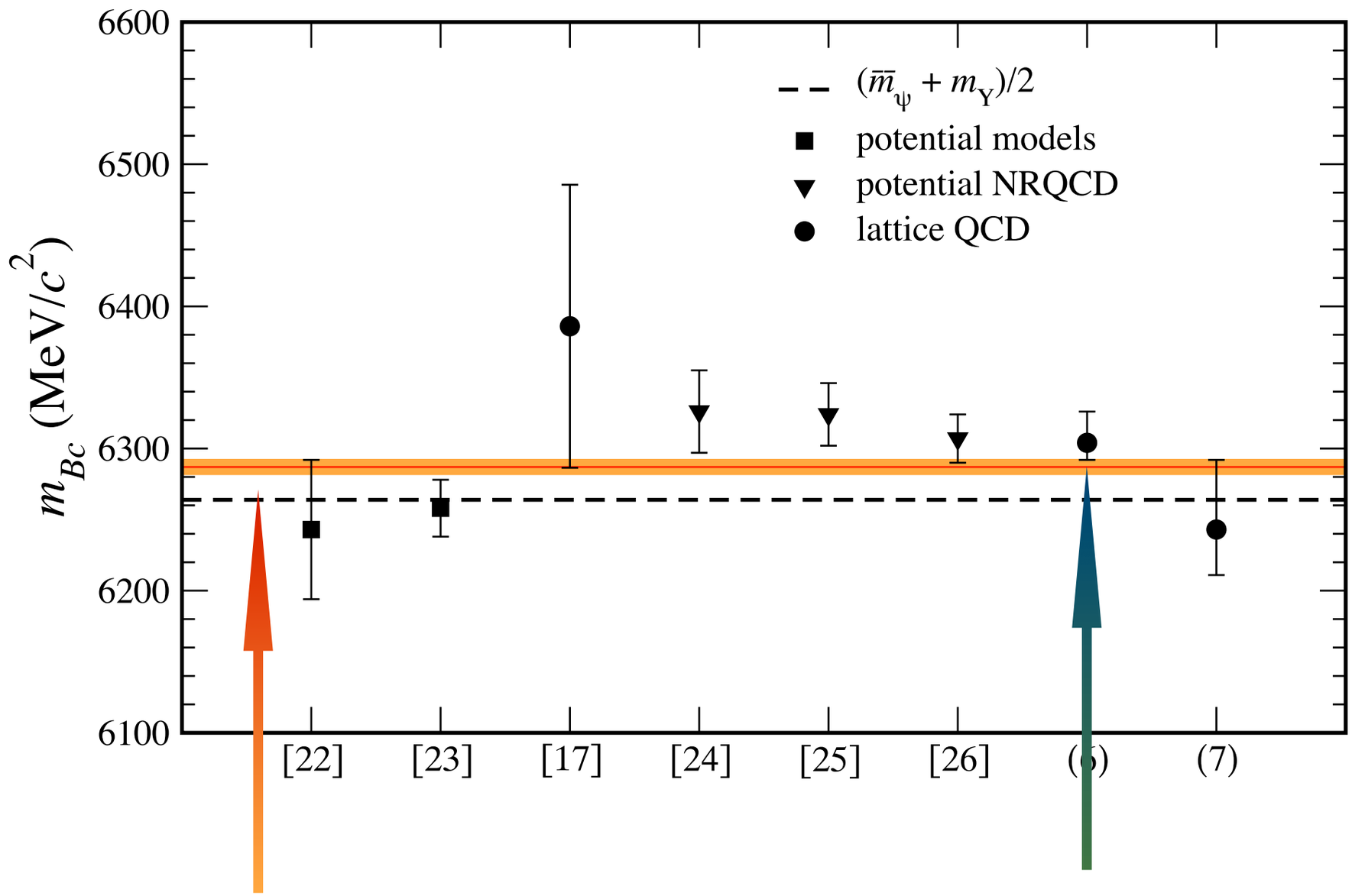
Models



Quenched lattice QCD.

Unquenched lattice QCD.

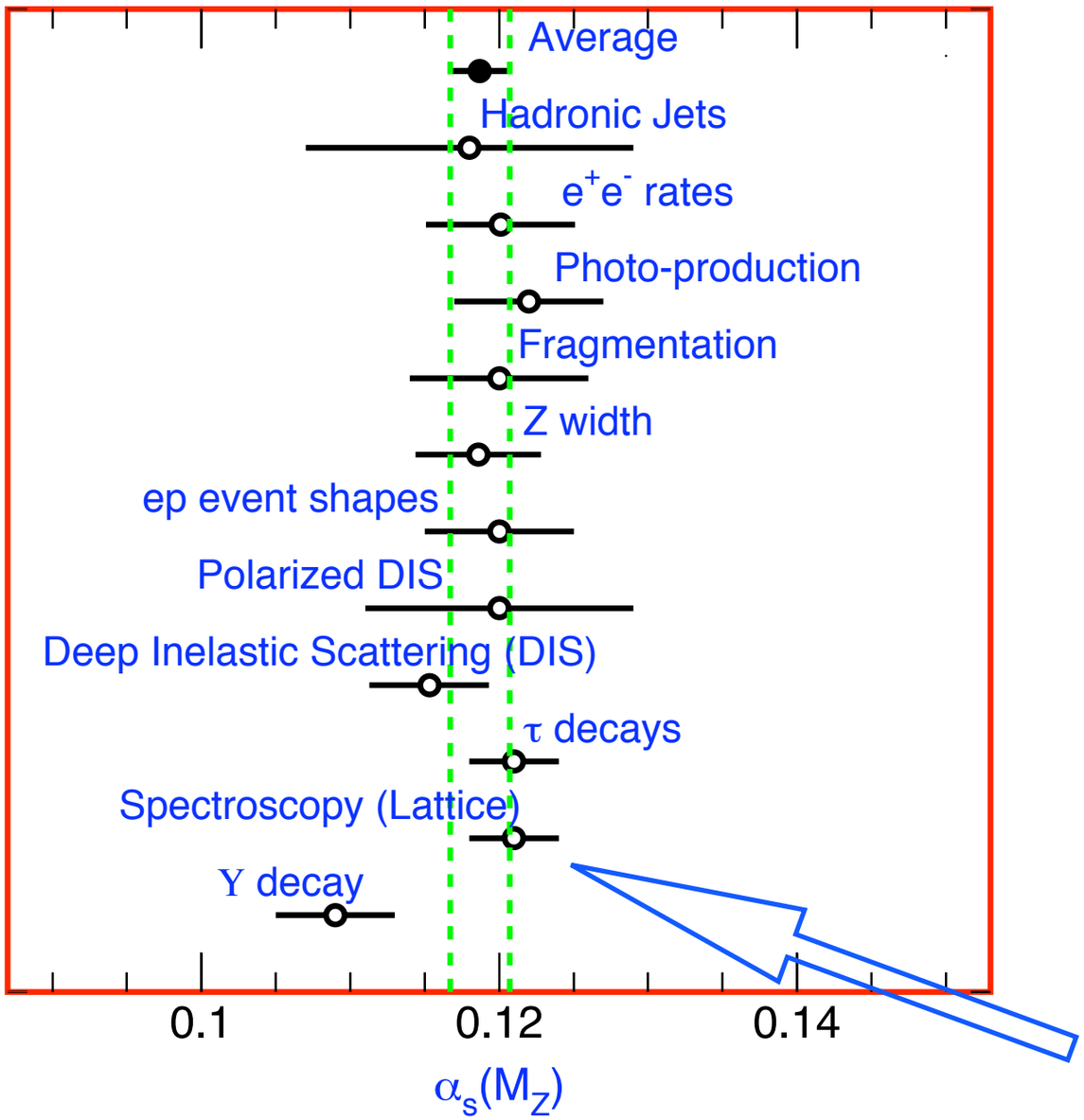
Experimental result (CDF December, 2004)



$m_{B_c} = 6287 \pm 5$ MeV
CDF, W&C seminar, 12/03/04

$m_{B_c} = 6304 \pm 12^{+18}_{-0}$ MeV
[hep-lat/0411027]

The strong coupling constant, α_s



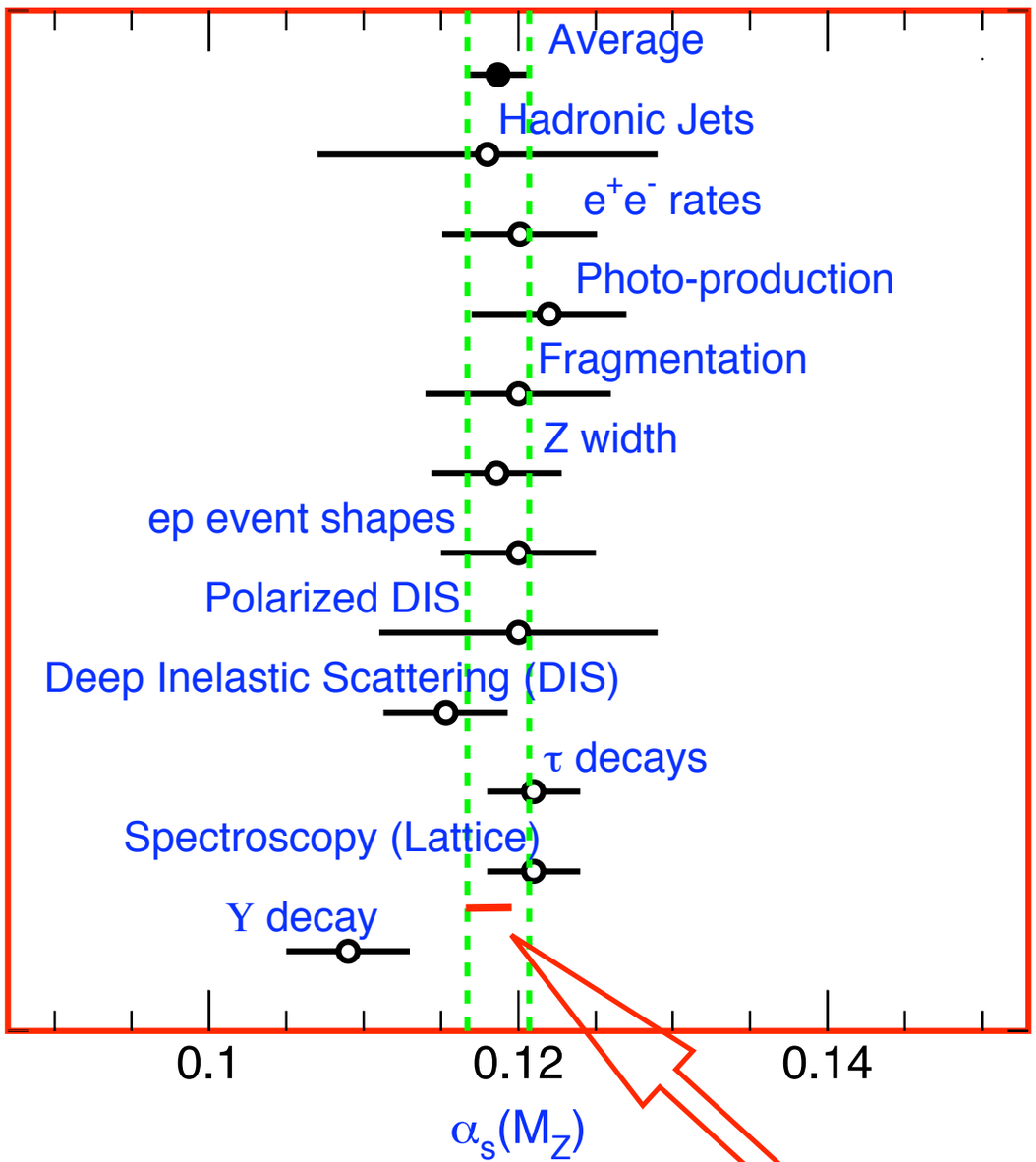
Particle Data Group, 2004.

Can be obtained from many high energy processes with perturbation theory.

On the lattice, tune the quark masses and strong coupling constant to reproduce observed hadron masses,
convert lattice coupling constant to continuum coupling constant.

Agrees! (Davies et al.)

The strong coupling constant, α_s



Can be obtained from many high energy processes with perturbation theory.

On the lattice, tune the quark masses and strong coupling constant to reproduce observed hadron masses,
convert lattice coupling constant to continuum coupling constant.

Mason et al., 2005.
Lattice QCD now gives the smallest errors.

Light quark masses, m_s and m_l

↑ $\equiv \frac{1}{2}(m_u+m_d)$

Only lattice QCD can obtain these from first principles in a systematically improvable way.

Old quark model guess: m_s = 150 MeV.

Wrong!

Lattice result:

$$
\begin{aligned}
m_s^{\overline{\rm MS}}(2\,{\rm GeV}) &= 76(0)(3)(7)(0)\ {\rm MeV} \\
\hat{m}^{\overline{\rm MS}}(2\,{\rm GeV}) &= 2.8(0)(1)(3)(0)\ {\rm MeV} \\
m_s/\hat{m} &= 27.4(1)(4)(0)(1)\ ,
\end{aligned}
$$

Obtain by matching lattice calculations of pion and kaon masses to experiment.

Aubin et al., 2004.

Most of the results shown have been for hadronically stable mesons, currently the cleanest lattice QCD calculations.

Almost all of the CKM matrix elements can be determined from such processes:

$$\begin{pmatrix} \frac{V_{us}}{f_\pi} & \frac{V_{us}}{f_K} & V_{ub} \\ & \frac{K \to \pi l \nu}{V_{cs}} & \frac{B \to \pi l \nu}{V_{cb}} \\ \frac{V_{cs}}{f_D} & \frac{V_{cs}}{f_{D_s}} & \\ & \frac{D \to K l \nu}{V_{ts}} & \frac{B \to K l \nu}{V_{tb}} \\ \frac{V_{ts}}{< B|\overline{B} >} & \frac{V_{ts}}{< B_s|\overline{B_s} >} & \end{pmatrix}$$

Example, $D \to Kl\nu$ semileptonic decay

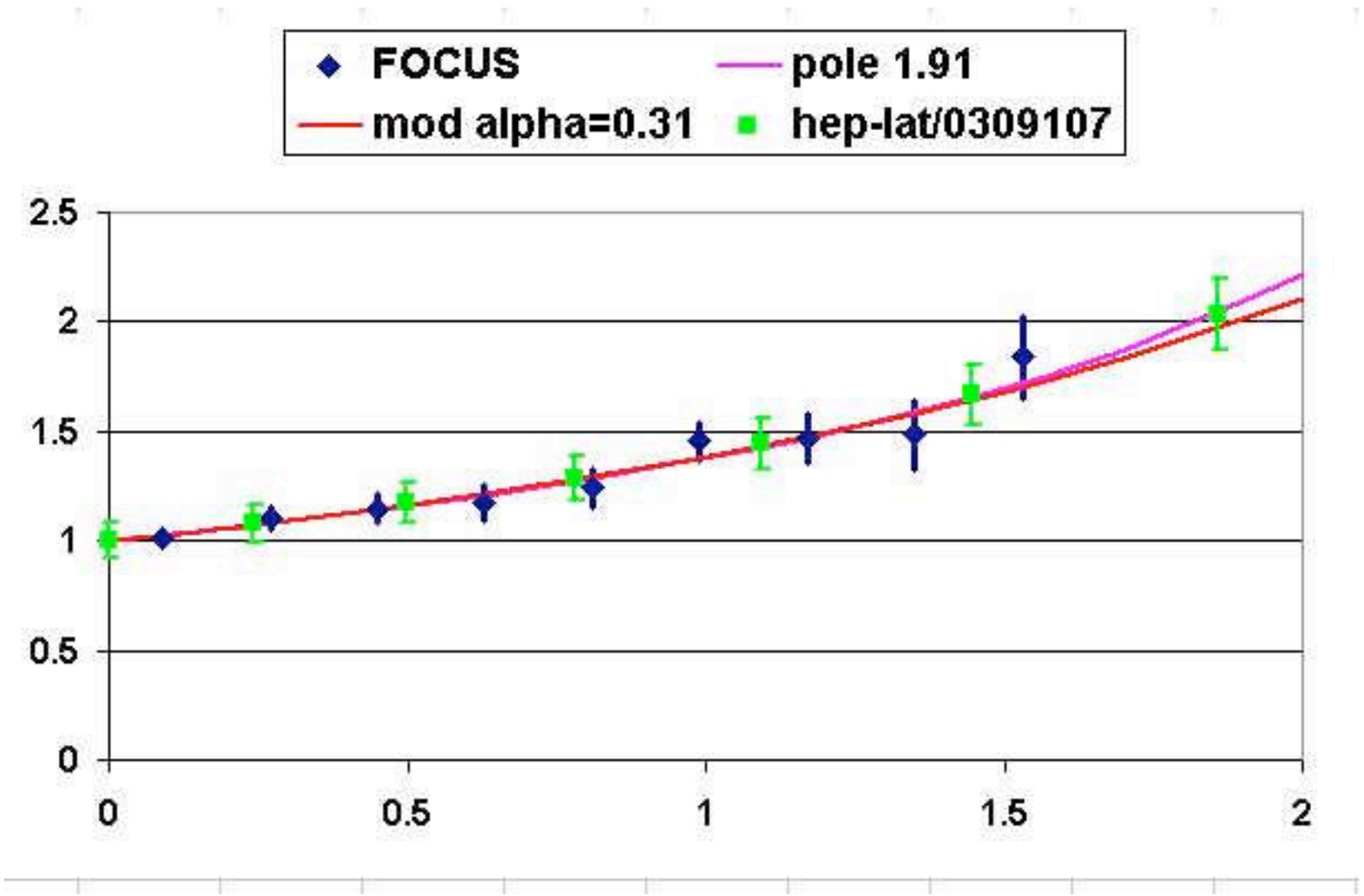
Theory and experiment for shape of form factor agrees.

Comparison to FOCUS experiment for $D \to K$ decay

$f_+(q^2)/f_+(0)$ vs q^2:

Green, lattice
(Fermilab/MILC)

Blue, experiment
(Focus)



Fermilab/MILC

(provided by FOCUS col

Comparing normalization of form factors gives CKM matrix elements.

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \mathrm{N/A} & \mathrm{N/A} & 3.6(5)(4)(3)\times 10^{-3} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ 0.24(1)(2)(2) & 0.97(4)(8)(2) & 3.8(1)(1)(6)\times 10^{-2} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \mathrm{N/A} & \mathrm{N/A} & \mathrm{N/A} \end{pmatrix}$$

Okamoto, 2004

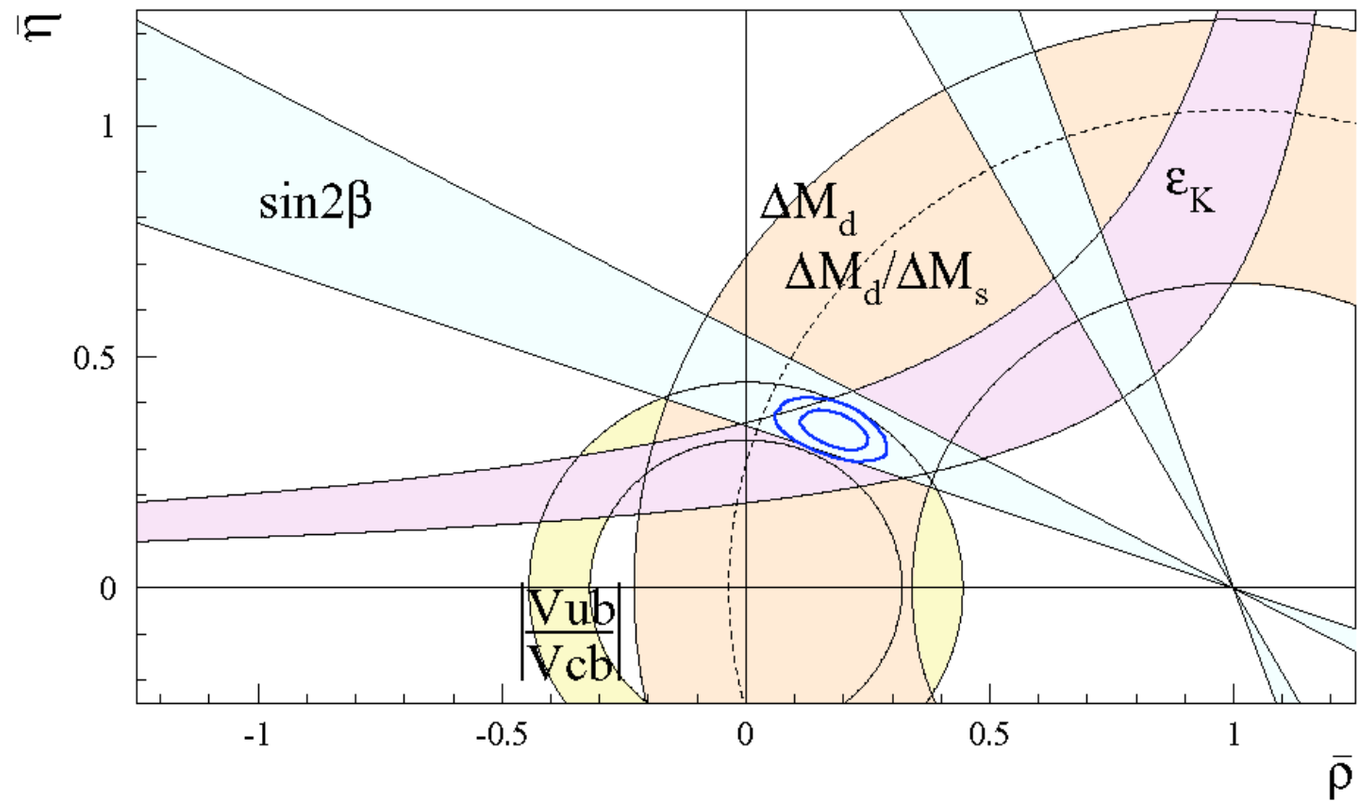
Check unitarity of second row:

$$(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(8)(2)$$

The ρ-η plane

$$\rho - i\eta \propto V_{ub}$$

Parameterizes CP violation in the Standard Model.



M. Ciuchini **hep-ph/0307195.**

The challenge:
Of the five best constraints to ρ and η,
most require lattice QCD to improve.

K K bar mixing currently dominated by lattice uncertainties.
B B bar currently dominated by lattice uncertainties.
Bs Bs bar will be dominated by lattice uncertainties.
Vub: (B→πlν) will be dominated by lattice uncertainties.
sin2β: dominated by experimental uncertainty.

The progress has been exciting, but the challenge ahead is large.

Summary

- There has been terrific progress in understanding the dynamics of quarks and gluons with quantitative first-principles calculations.
  - Some simple things are now done.
  - Many other important things look doable.
- A rich array of further challenges awaits in QCD and beyond the Standard Model physics.