Current Trends and Phenomena in the Analysis of Nonlinear Partial Differential Equations

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Abstract

The mini-symposium focuses on current trends and phenomena in the analysis of nonlinear partial differential equations (PDEs). The main theme is to gather experts working in diverse fields of PDEs to discuss on some recent results in existence, regularity and long time dynamics of solutions. The mini-symposium is also a forum to propose new theory and methods in the analysis and applications of nonlinear PDEs. The equations considered in this mini-symposium arise from both theoretical and applied sciences. They may include nonlinear Schrödinger, nonlinear diffusion, wave, Navier-Stokes, Ohta-Kawasaki equations.

Session 1

- Borislav Yordanov (yordanov@math.utk.edu), Department of Mathematics, University of Tennessee, Knoxville, TN.

  Title:*On the Diffusion Phenomenon of Dissipative Wave Equations*

  Abstract: We discuss the diffusion phenomenon of dissipative wave equations in $\mathbf{R}^n$ or exterior domains and its generalization to dissipative evolution equations in a Hilbert space H. Given a nonnegative self-adjoint operator A with a dense domain $\mathcal{D}(A) \subset H$, we study the long time behavior of equations like

  $$\begin{aligned} &u''(t) + Au(t) + u'(t) = 0, \quad t \in (0, \infty), \\ &u(0) = u_0, \quad u'(0) = u_1 \end{aligned}$$

  and

  $$\begin{aligned} &u''(t) + Au(t) + Au'(t) = 0, \quad t \in (0, \infty), \\ &u(0) = u_0, \quad u'(0) = u_1, \end{aligned}$$

  where $(u_0, u_1) \in H \times \mathcal{D}(A^{1/2})$. We find asymptotics for $u(t)$ in terms of the semigroup e^{-tA} as $t \to \infty$.

  This talk is based on my joint work with professor Grozdena Todorova (todorova@math.utk.edu), UT-Knoxville.

- Hoai-Minh Nguyen (hmnguyen@umn.edu), School of Mathematics, University of Minnesota, MN.

  Title: *Some new characterizations of Sobolev spaces*

  Abstract: This talk is on some new characterizations of Sobolev spaces which are based on non-local functionals whose roots are from definition of the fractional Sobolev spaces. New types of the Poincare inequality, the Sobolev inequality, and the Reillich Kondrachov compactness theorem will also be discussed. Possible applications for image processing will be mentioned. This is partially joint with Bourgain and Brezis.

- Fabio Pusateri (fabiop@math.princeton.edu), Department of Mathematics, Princeton University.

  Title: *Fractional Schrödinger equations and water waves in two dimensions.*

  Abstract: We will discuss the problem of global existence and scattering for gravity water waves in 2 spatial dimension (one dimensional interfaces). In particular we will focus on an important connection with a simplified model, given by a cubic fractional NLS equation. We will give some details about the proof of global existence of solutions for this simpler model. Joint work with Alex Ionescu.

- Grozdena Todorova (todorova@math.utk.edu), Department of Mathematics, University of Tennessee, Knoxville, TN.

  Title:*On the Regularizing Effect of Nonlinear Damping in Hyperbolic Equations*

  Abstract: Global well-posedness in $H^2(R^3) \times H^1(R^3)$ is proved for nonlinear wave equations of the form
  $\Box u + f(u) + g(u_t) = 0$, where $t \in R_+,\ x \in R^3$. The main assumption is that the nonlinear damping $g(u_t)$ behaves like $|u_t|^{m-1}u_t$ with $m \geq 2$ and defocusing nonlinearity is like $f(u) = |u|^{p-1}u$ for any $p \geq 2$. This extends the result of Lions and Strauss for $H^2(R^3) \times H^1(R^3)$ well-posedness where the requirement is $m \geq p \geq 2$. The result also applies to certain exponentially growing functions, such as $f(u) = \sinh u$. We observe that the nonlinear damping gives rise to a new monotone quantity involving the second order derivatives of u which is an important part of global *a priori* estimates for initial data of any size.

  Global well-posedness in $H^1(R^3) \times L^2(R^3)$ is shown for the same equation in the critical case $f(u) = u^5$ and $g(u_t) = |u_t|^{2/3}u_t$. The main tool is a monotone quantity involving the first order derivatives of u.

  This talk is based on my joint work with professor Borislav Yordanov (yordanov@math.utk.edu), Department of Mathematics, University of Tennessee.

Session 2

- Hantaek Bae (hantaek@math.ucdavis.edu), Department of Mathematics, UC Davis, LA

  Title: *A priori estimates of the free boundary problem of the Navier-Stokes equations with surface tension.*

  Abstract: In this talk, we explain how to obtain the H^2 bound of the velocity field of the Navier-Stokes equations with surface tension. The main idea is to extend the boundary terms into the fluid domain, which allows us to identify a correct differential operator $\mathscr{A}$. This differential operator behaves like the Laplacian on the whole spaces so we can minimize the effect of the free boundary when we perform a priori estimates to estimate the velocity field in H^2.

- Truyen Nguyen (tnguyen@uakron.edu), Department of Mathematics, University of Akron, OH.

  Title:*On the regularity of optimal maps arising in optimal transportation*

  Abstract: We study interior regularity for optimal maps of optimal transportation problems with power costs. We show that our problems are equivalent to a new optimal transportation problem whose cost function is a sufficiently small perturbation of the quadratic cost. We establish that the second derivatives of the potential function of the optimal map are Hölder continuous. Our proof consists in a perturbation argument from the standard Monge-Ampère equation and some quantitative estimates on how the equation degenerates near the boundary.

- Robert McCann (mccann@math.toronto.edu), Department of Mathematics, University of Toronto, Toronto, Canada.

  Title: *Optimal transportation with capacity constraints* (part 1)

  Abstract: The classical problem of optimal transportation can be formulated as a linear optimization problem on a convex domain: among all joint measures with fixed marginals find the optimal one, where optimality is measured against a given cost function. Here we consider a variation of this problem by imposing an upper bound constraining the joint measures, namely: among all joint measures with fixed marginals and dominated by a fixed measure, find the optimal one. After computing illustrative examples, we given conditions guaranteeing uniqueness of the optimizer and initiate a study of its properties. Based on a preprint arXived with Jonathan Korman.

- Robert McCann (mccann@math.toronto.edu), Department of Mathematics, University of Toronto, Toronto, Canada.

  Title: *Optimal transportation with capacity constraints* (part 2)

  Abstract: The classical problem of optimal transportation can be formulated as a linear optimization problem on a convex domain: among all joint measures with fixed marginals find the optimal one, where optimality is measured against a given cost function. Here we consider a variation of this problem by imposing an upper bound constraining the joint measures, namely: among all joint measures with fixed marginals and dominated by a fixed measure, find the optimal one. After computing illustrative examples, we given conditions guaranteeing uniqueness of the optimizer and initiate a study of its properties. Based on a preprint arXived with Jonathan Korman.