Multilevel Monte Carlo Method with Application to Uncertainty Quantification in Oil Reservoir Simulation

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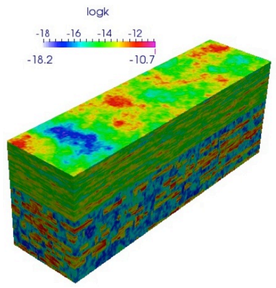
Problem Statement

Uncertainty quantification in reservoir simulation



- A 3-D reservoir model with size 60*220*85.
- The permeability field is strongly heterogeneous.
- Quantification of the influence of the permeability uncertainty on oil and gas production is very computationally expensive.

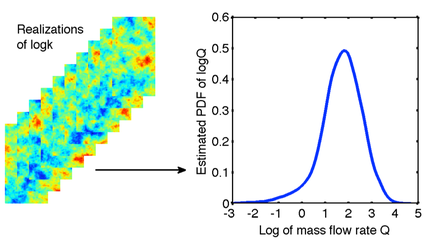
❖ The rational management of oil and gas reservoir requires an understanding of its response to existing and planned schemes of exploitation and operation.

❖ Such understanding requires analyzing and quantifying the influence of the subsurface uncertainty on predictions of oil and gas production.

❖ In uncertainty quantification, the commonly used stochastic techniques like moment equation methods, generalized polynomial chaos expansions and stochastic collocation methods are not suitable for the strongly heterogeneous reservoir problem.

❖ The dimension independent Monte Carlo simulation becomes the choice.

Monte Carlo simulation



- Based on measurements of permeability k, random realizations of log$k$ are generated.
- For each realization of log$k$ field, simulate the reservoir model to get simulations of oil production Q.
- Summarize simulated results of Q statistically to quantify the influence of log$k$ uncertainty on Q.

Computational cost

❖ The standard MC simulation is very computationally expensive because
  - a large number of model executions is required to achieve convergence;
  - each model execution is time costly simulated on a fine spatial grid to ensure accuracy.

❖ A computationally efficient method is desired to quantify the uncertainty of the strongly heterogeneous reservoir problem.

Standard Monte Carlo Method

We are interested in estimating the expectation E[Q] of predicted oil production Q by simulating Q at the numerical model with size M.

The standard MC estimator for E[Q_M] is

$$E[Q_M] \approx \hat{Q}_M^{MC} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} Q_M^{(i)}$$

The mean square error (MSE) of the estimator is

$$e\left(\hat{Q}_M^{MC}\right)^2 = E\left[\left(\hat{Q}_M^{MC} - E[Q]\right)^2\right]$$

$$= N_{MC}^{-1} V[Q_M] + \left(E[Q_M - Q]\right)^2$$

❖ First term is variance of the MC estimator, which is small as V[Q_M] is small and decays inversely with the number of samples N_{MC}.

❖ Second term is square of the error in expectation between Q_M and Q, which can be reduced by using a fine numerical grid.

Multilevel Monte Carlo Method

The idea of MLMC can be formulated as:

$$E[Q_M] = E[Q_{M_0}] + \sum_{l=1}^{L} E[Q_{M_l} - Q_{M_{l-1}}] = \sum_{l=0}^{L} E[Y_l]$$

where l=0,1,…,L is levels, and model grid size $M_0<M_1<\ldots<M_L=M$.

$$E[Y_0] \approx \hat{Y}_0 = \frac{1}{N_0} \sum_{i=1}^{N_0} Q_0^{(i)} \text{ and } E[Y_l] \approx \hat{Y}_l = \frac{1}{N_l} \sum_{i=1}^{N_l} (Q_l^{(i)} - Q_{l-1}^{(i)})$$

The MLMC estimator for E[Q_M] is

$$E[Q_M] \approx \hat{Q}_M^{ML} = \sum_{l=0}^{L} \hat{Y}_l$$

The MSE of the MLMC estimator is

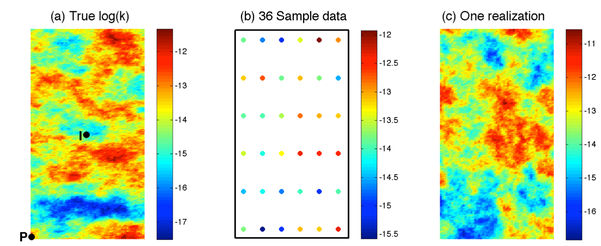
$$e\left(\hat{Q}_M^{ML}\right)^2 = E\left[\left(\hat{Q}_M^{ML} - E[Q]\right)^2\right]$$

$$= \sum_{l=0}^{L} N_l^{-1} V[Y_l] + \left(E[Q_M - Q]\right)^2$$

❖ First term reflects the advantage of MLMC estimator achieving the same accuracy as standard MC but with less computational cost.

Application to a Reservoir Model

Model description



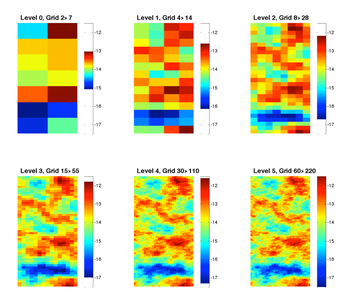
(a) True permeability field log(k); I: injection well; P: production well

(b) 36 sample data conditioned to generate log(k) realizations

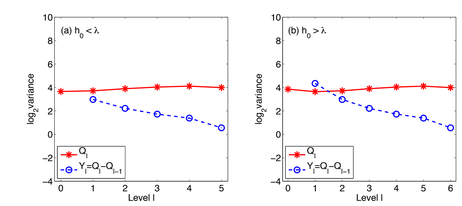
(c) One realization of log(k) field

We are interested in estimating oil production E[Q] at location P.

Six levels used in MLMC

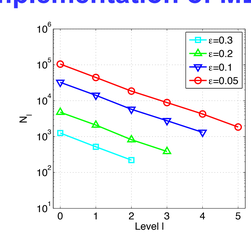


Variance decays with levels



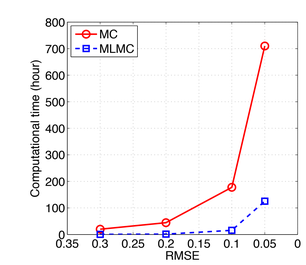
- h_0 is cell length of the coarsest level; λ is correlation length.
- The coarsest level is determined as $h_0 < \lambda$.
- Variance decreasing with levels suggests fewer samples are needed on computationally costly higher levels.

Implementation of MLMC



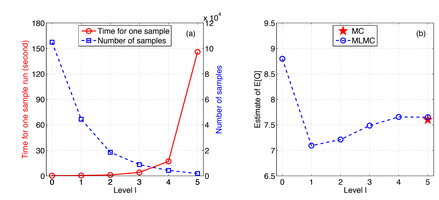
- The number of levels and the number of samples in each level are determined based on users' specified RMSE accuracy ε.

Computational efficiency of MLMC



- To achieve the same RMSE, MLMC needs significantly less computational time;
- For the same computational time, MLMC can achieve higher accuracy with smaller RMSE.

Analysis of MLMC efficiency



- MLMC run more samples on computationally frugal coarse grids and a few samples on computationally expensive fine grids;
- MLMC estimate approaches MC estimate as more levels considered.

Conclusions

❖ MLMC method can evaluate uncertainty in strongly heterogeneous reservoir problems efficiently and effectively.

❖ MLMC method is model independent and flexible to be used together with any MC estimators.