Householder Fellowship

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Summary

The Householder Fellowship provides new computational scientists with opportunities to pursue research in pure and applied mathematics in an area related to high performance scientific computing and in a scientific area aligned to the missions of the Department of Energy. Post-processing techniques for discontinuous Galerkin (DG) methods for linear hyperbolic equations have been developed under funding from this Fellowship.

The discontinuous Galerkin method is increasingly used in many applications because of its ability to handle complex geometries, simple treatment of boundary conditions, high order accuracy, flexibility for adaptivity, and because it can easily be adapted to run on a parallel machine. To fully utilize the potential of the inherent high order accuracy, a post-processing technique has been developed. This post-processing technique has proven useful for time-dependent linear hyperbolic equations and essentially doubles the order of accuracy over uniform meshes from k+1 of the discontinuous Galerkin method to 2k+1 for the post-processed solution (where k is the highest degree polynomial used in the approximation). The post-processor is local, and therefore relies only on information from its element neighbors. The post-processor can be implemented using very efficient small matrix-vector calculations. In Figures 1 and 2, the advantage of post-processing the discontinuous Galerkin solution is apparent with the nearly ten fold reduction in error for the linearized Euler equation. This post-processing technique is most effective on uniform meshes and if the region of the discontinuity is known *a priori*. Determination of the appropriate post-processing stencil in the neighborhood of a discontinuity where the location is not known is a difficult task. Furthermore, another challenge in implementing this post-processing technique is the extension to smoothly varying meshes.

We borrow ideas from Essentially Non-Oscillatory (ENO) methods as well as local edge detection methods to determine the region of discontinuity and the appropriate local post-processing stencil. Both methods are effective for finding the appropriate post-processing stencil and correctly choosing centered post-processing in smooth regions and one-sided or partially one-sided post-processing in the neighborhood of a discontinuity. The expected 2k+1 order accuracy is achieved throughout the entire domain as long as the discontinuity lies on an element boundary. This is an important tool for use not only in choosing the appropriate post-processing stencil, but can also be used to determine regions where limiting is necessary. The ease of

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implementation makes it ideally suited for a variety of applications where discontinuity detection is essential. Further details can be found in [1].

A second important accomplishment in the area of post-processing for discontinuous Galerkin methods is extension to smoothly varying meshes. The current proofs for this method are for time-dependent linear hyperbolic equations over a locally uniform mesh. In order to extend this technique to smoothly varying meshes, a simple local projection to a uniform mesh before post-processing is implemented. This allows us to obtain the expected $2k+1$ order accuracy in the post-processed solution. This approach shows promise for other applications on more complicated mesh structures.

[1] J.K. Ryan, R. Archibald, A. Gelb, S. Gottlieb, "One-sided post-processing for the discontinuous Galerkin method using ENO type stencil choosing and the local edge detection method," *Journal of Scientific Computing*, to appear.

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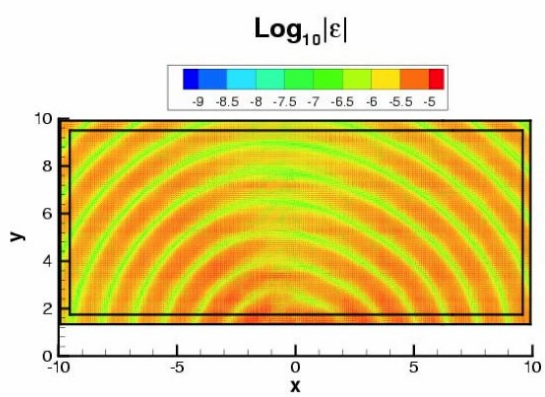

Figure1: Contour plot of the errors of the linearized Euler equations using the discontinuous Galerkin method without post-processing.

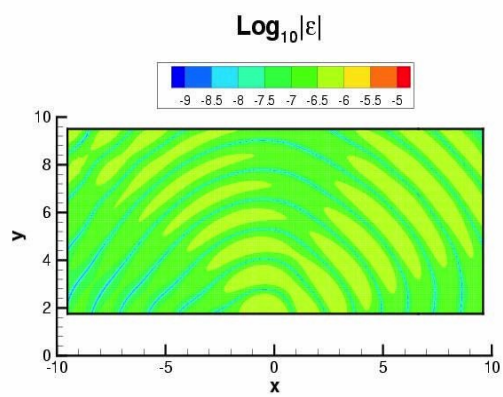

Figure2: Contour plot of the errors of the linearized Euler equation using the discontinuous Galerkin method with post-processing. The magnitude of the error has significantly improved.

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