A Cubic Hermite Boundary Integral Approximation

L. J. Gray*, Oak Ridge National Laboratory
M. Garzon, University of Oviedo, Oviedo Spain

Summary

A differentiable (cubic Hermite) boundary integral interpolation has been developed. This approximation should be particularly effective for the solution of moving boundary problems.

A primary motivation for this work is to develop an effective coupled Level Set and boundary integral algorithm for the numerical solution of moving boundary problems. Boundary integral methods should be advantageous for this important class of problems, as re-meshing the evolving boundary is easier than re-meshing the entire volume. Moreover, determining the surface velocity generally requires computing the gradient of the primary function on the boundary, and integral equation methods are capable of producing highly accurate derivatives.

There are many important advantages of the Level Set method for tracking interfaces. An additional benefit is that, in addition to the location of the interface, the Level Set function can also provide geometric information (surface normals). The goal of this work is to allow the boundary integral analysis to exploit this extra information. The knowledge of surface normals will permit a differentiable interpolation of the boundary and of the primary function, resulting in a much more accurate integral equation solution.

To accomplish this, a cubic Hermite approximation for two-dimensional boundary integral analysis has been developed. The method differs from previous Hermite interpolation algorithms in that (a) the gradient equations are sparse, significantly reducing the computational cost, and (b) the surface normal information is incorporated to achieve a smooth cubic interpolation of the geometry. It is expected that these results can be extended to three dimensions.

This technique has been tested using a nonlinear potential flow model for wave breaking. The figure below shows results for a wave where the ocean bottom is sinusoidal.

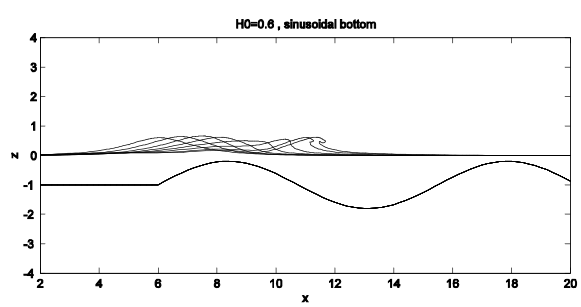

Figure 1. Coupled Level Set/boundary integral simulation of wave breaking

* (865) 574-8189, ljg@ornl.gov

[1] M. Garzon and L. J. Gray, “On a Hermite Boundary Integral Approximation,” Invited paper for special issue of *Computers and Structures* (in press).

For further information on this subject contact:
Gary Johnson, Program Manager
Mathematical, Information, and Computational Sciences Division
Office of Advanced Scientific Computing Research
Phone: 301-555-3691
garyj@er.doe.gov