Galerkin Hypersingular Integration

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Summary

Algorithms for the direct evaluation of hypersingular boundary integrals have been developed. Although individual element integrals are divergent, it has been proven that the complete boundary integration is finite. The key advantages of the direct approach are that it is general --- applicable to an arbitrary Green's function -- and that it forms the basis for efficient gradient evaluation methods.

Despite the fact that hypersingular equations are a well established and essential component of boundary integral analysis, basic issues -- the definition and evaluation of hypersingular integrals -- remain difficult. For a Galerkin formulation, singular integration is most commonly accomplished by employing Stokes Theorem to reformulate the equation, reducing the order of the singularities. While successful, the main drawback of this approach is that for complicated Green's functions (e.g., functionally graded materials), even if the integration by parts is possible, the numerical implementation can be quite onerous.

We have developed non-Stokes direct algorithms based upon defining the (hyper)singular integrals as boundary limits [1]. This approach provides a sound mathematical definition of the integrals, and moreover establishes that the integral is finite, even though separate integrations over individual elements are shown to be divergent. Moreover, as the algorithms are based upon analytic evaluation at the singular point, the accuracy is excellent.

There are two important aspects of the new algorithms, the first being that they are applicable to a general Green's function. As an example, implementation for a thermal analysis in a functionally graded material has been carried out [2]. In addition, preliminary work [3] suggests that the methods will prove equally successful for the especially difficult Green's function for anisotropic elasticity. In this case, the fundamental solution is not known in closed form, and thus new techniques are needed to invoke the analytic integration and limit process.

The second significant advantage is that the limit definition leads to a highly efficient method for computing all function derivatives on the boundary [4]. This is a key step in many applications, most particularly for 'moving boundary' simulations.

As an example, a Galerkin boundary integral analysis has been employed to solve for the steady state temperature distribution in a graded compressor blade. The thermal conductivity varies exponentially in the z

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direction. These results (see Fig. 1) compared well to a corresponding finite element analysis.

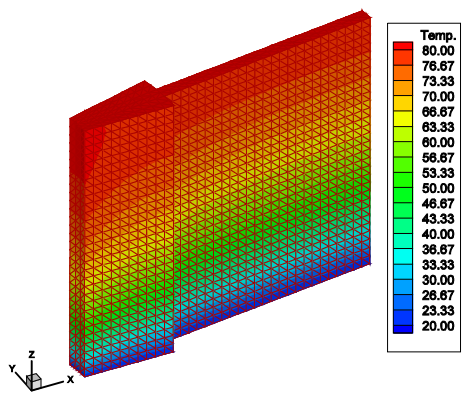

Fig. 1. Contour plot of the temperature distribution in a graded compressor blade.

[1] L. J. Gray, J. M. Glaeser, and T. Kaplan, "Direct Evaluation of Hypersingular Galerkin Surface Integrals," *SIAM J. Scientific Computing* (in press)

[2] A. Sutradhar, G. H. Paulino, and L. J. Gray, "Symmetric Galerkin boundary element method for heat conduction in functionally graded materials," *Int. J. Num. Meth. Eng*. (submitted)

[3] L. J. Gray, A. Griffith, L. Johnson and P. A. Wawrzynek, "Evaluation of Galerkin Singular Integrals for Anisotropic Elasticity: Displacement Equation," *EJEE* (in press)

[4] L .J. Gray, A.-V. Phan, and T. Kaplan, "Boundary Integral Evaluation of Surface Derivatives," *SIAM J. Scientific Computing* (submitted)

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