Boundary Integral Analysis for Functionally Graded Materials

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Summary

Fundamental solutions (Green's functions) to the partial differential equations in a functionally graded material (FGM) have been derived. Based upon these exact solutions, effective numerical methods for solving boundary integral equations can be applied to important engineering problems, e.g., fracture analysis, and material design optimization.

Functionally graded materials (FGMs) are a class of relatively new and promising materials that have emerged from the need to optimize material performance. In a homogeneous material the properties are constant, whereas in an FGM, the material properties vary continuously with position, usually in one coordinate direction.

As an example, a material that transitions smoothly from a pure metal to a pure ceramic would combine the high temperature and compressive strength properties of the ceramic with the fracture toughness and thermal conductivity of the metal. Compared to a layered system, e.g., a ceramic coating on a metal substrate, an FGM avoids the discontinuity in material properties across the interface. A sharp interface can lead to stress concentrations that eventually produce material failure. FGMs have already been employed in many important areas, e.g., thermal barrier coatings for aerospace applications, graded refractive index materials for optical applications, and implants for bio-medical applications.

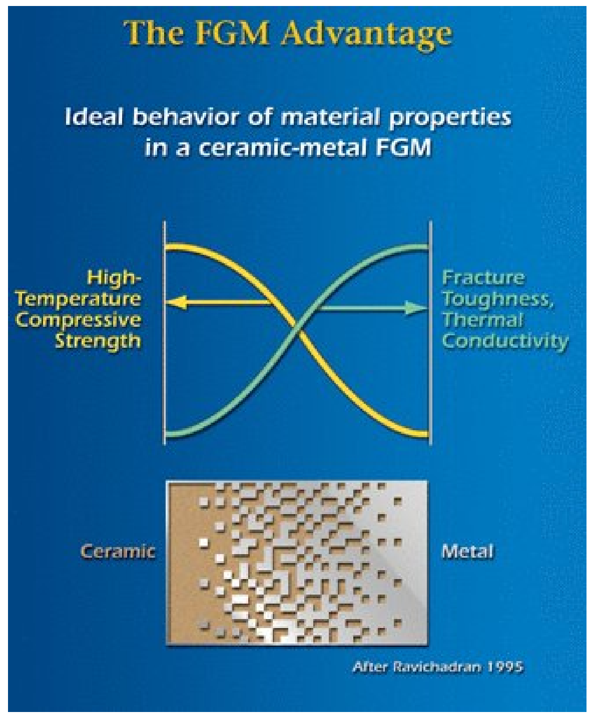

Figure 1. The FGM Advantage.

As analytical solutions for non-homogeneous materials are rare, there is a need for reliable and efficient numerical methods for solving problems in FGMs. In particular, a critical engineering need, and a primary goal of this work, is to understand failure in FGMs, i.e., crack propagation

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studies. For homogeneous materials, integral equation methods have two significant advantages for these analyses. First, the re-meshing required as the crack grows is only at the crack tip, and is therefore (compared to volume methods such as finite elements (FEM)) relatively trivial. Second, unlike FEM, the boundary-only approach avoids a direct confrontation with the stress singularity ahead of the crack. As a consequence, an appropriate crack tip element [1] produces highly accurate stress intensity factors (which determine crack growth), even with seemingly coarse meshes. Thus, the overall computation time, including mesh generation, for a boundary integral analysis is likely to be significantly less than for an FEM simulation having corresponding accuracy.

Integral equation methods, however, are based upon knowing the **Green's function**, and have therefore been limited almost exclusively to homogeneous materials. The Green's function is an exact solution of the partial differential equations, one having an impulse forcing term at one point, and are known only for the simplest equations.

We have successfully derived the Green's function for an FGM wherein the material properties vary exponentially through the solid. The Green's function has been obtained in closed form for the Laplace equation (heat conduction) [2,3], and in a readily computable form for elasticity [4,5] in both two and three dimensions. With these Green's function, an FGM analysis can be formulated in terms of boundary integral equations as simply as for homogeneous media. Note that the advantages of a boundary integral formulation for fracture analysis are amplified in the FGM setting. For an FGM, the FEM volume mesh must be fine enough to represent the exponential grading in the material; this adds to the computation cost and further complicates automatic re-meshing schemes. In the boundary integral approach, the material gradation in the volume is exactly encapsulated in the Green's function.

The ability to perform computational analyses is highly important for FGM research and development, as the manufacturing and experimental testing of these materials is in its infancy and is not simple nor inexpensive. In particular, parameter design optimization and crack propagation are two areas of major importance for which numerical simulations can be highly useful.

[1] L. J. Gray, A.-V. Phan, G. H. Paulino, and T. Kaplan, "Improved Quarter-Point Crack Tip Element," *Engineering Fracture Mechanics* **70**, pp. 269-283 (2003).

[2] L. J. Gray, T. Kaplan, J. D. Richardson, and G. H. Paulino, "Green's Function and Boundary Integral Analysis for Exponentially Graded Materials: Heat Conduction," *ASME Journal of Applied Mechanics* (in press).

[3] A. Sutradhar, G. H. Paulino, and L. J. Gray, "Symmetric Galerkin boundary element method for heat conduction in functionally graded materials," *Int. J. Num. Meth. Eng*. (submitted)

[4] P. A. Martin, J. D. Richardson, L. J. Gray, and J. Berger, "Green's functions for an Exponentially Graded Elastic Material," *Proc. Royal Soc*. **458**, 1931-1948 (2002)

[5] Y.-S. Chan, L. J. Gray, T. Kaplan, and G. H. Paulino, "Green's Function for a Two-Dimensional Exponentially-Graded Elastic Medium," *Proc. Royal Soc.* (submitted)

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