Mesh Generation for 3D MHD Stability Analyses

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Summary

In many physics and engineering computational modeling applications the geometry of the model is often produced as part of the numerical solution itself. In addition, interfacial surfaces obtained from physics-based simulation frequently undergo an optimization process as the solution evolves. One particular application is the design and optimization of advanced, three-dimensional (3D) fusion confinement devices known as stellarators. This simulation process results in the geometric evolution of the model, which requires the mesh to be generated at every step of computation. However, the process of mesh generation can be non-trivial due to the complexity of the geometric model and mesh constrains required by the physics. At the computational mathematics group, we developed a robust mesh generation system that is used to construct orthogonal smooth meshes as the solution of the model evolves. The generation system guarantees the mesh in use conations non-vanishing or non-negative Jacobains. It can be linked to the physics code to run automatically with no human intervention.

Volume Mesh Optimization for Fusion Simulation

The design of advanced, three-dimensional (3D) fusion confinement devices known as stellarators involves, among other things, the evaluation of stability criterion based on the linearized ideal magnetohydrodynamic (MHD) equations. This is a computationally intensive task that is performed numerically many times inside of an optimization loop to identify stellarator configurations with suitable physical and engineering properties. Numerical 3D equilibria with toroidally-nested magnetic flux surfaces form the basis for these calculations. Within the boundary separating confined plasma (with nonzero kinetic pressure) from a vacuum (zero pressure) region, the equilibrium solution is described by computing an inverse parametric representation of the cylindrical coordinates (R, ϕ, Z) in terms of magnetic flux coordinates (s, θ, φ) as follows:

$$R = \sum R_{mn}(s)\cos(m\theta + n\varphi)$$
$$Z = \sum Z_{mn}(s)\sin(m\theta + n\varphi)$$
$$\phi = \varphi$$

The parameter s labels the magnetic surfaces, with s = 1 denoting the plasma/vacuum interface. The Fourier expansion coefficients are computed by solving a set of non-linear elliptical equations for the MHD equilibrium $J \times B = \nabla p$. These inverse expressions can be used to map out a non-singular set of generally non-orthogonal flux coordinates that can be used for computing the magnetic energy perturbations inside the plasma (s < 1). Evaluating the MHD stability for external modes (kink modes and vertical displacements) requires computing the magnetic energy perturbations in the vacuum region, between the plasma boundary (s = 1) and a conducting shell located at some distance - not necessarily uniform around the surface –

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measured as a small multiple (typically not more than 2-3) of the mean minor radius of plasma.

It is therefore necessary to extend this coordinate system into the vacuum region in order to evaluate the kink stability for the stellarator. This is non-trivial due to the complexity (strong toroidal variation) and 3D character of the boundary. Moreover, it is required that the coordinates make a smooth transition at the plasma vacuum interface (s=1), since both angular and radial (s) derivatives of the dependent magnetic field quantities and displacement vectors need to be evaluated on the resulting mesh. A simple continuation of the plasma coordinate system often results in the Jacobian changing sign in the vacuum region. Thus, a more robust method of vacuum mesh generation is required to perform the stability analysis of 3D plasma equilibria.

Previously, the mesh was generated between the outer and inner surfaces by means of algebraic interpolation method. Due to the 3D twist in the prescribed surface geometry, the resulting volume mesh contained degenerate mesh cells caused by mesh collapse, folding, or overlaps. This resulted in the evaluation of vanishing or negative volume mesh cells that renders the mesh unsuitable for direct use further computation of the physics equations. In such a case, human intervention is required to untangle the mesh using stand alone meshing tools.

We have developed a robust elliptic mesh generator based on quasi-conformal mapping for geometric complex regions. This system enables the user to generate meshes on the surfaces and in the vacuum region bounded by the interfacial surfaces as well, with non-vanishing and non-negative volumes. The system can be used to generate or to optimize an existing mesh at every step of simulation process. The advantages of this system over using just algebraic interpolation methods are the automation and the coupling of the system to the physics code. Other advantages are the use of the system to construct orthogonal and smooth meshes. These geometric criteria can be essential for the convergence and the accuracy of the solution. Presented below in Figure 1 are two view of an orthogonal smooth volume mesh in the vacuum region generated using our mesh generator for used to evaluate the kink stability for the stellarator.

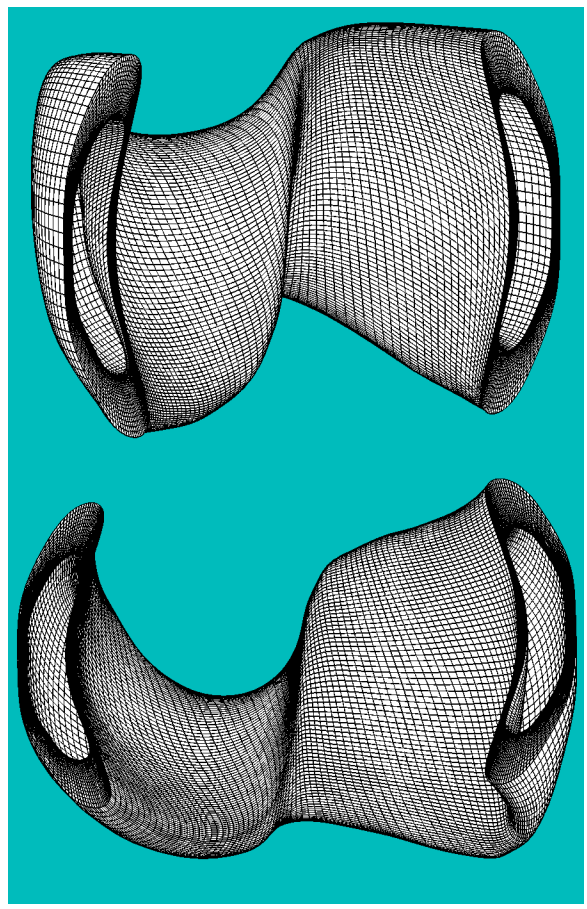

Figure 1. ***Orthogonal Smooth Volume Mesh for the Stellarator Configuration***

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