

External view

[M]

Ext	$in, out : \text{seq } M$
	$\exists s : \text{seq } M \bullet in = s \wedge out$

Initially

$$\text{ExtInit} \triangleq [\text{Ext}' \mid in' = \langle \rangle]$$

Transmit

ΔExt	
$m? : M$	
$in' = \langle m? \rangle \wedge in$	
$out' = out$	

Sectional view

[SPC]

$Receive$	ΔExt
$in' = in$	
$#out' = #out + 1 \vee out' = out$	

$[X]$
$_ \simeq _ : \text{seq}(\text{seq } X) \times \text{seq}(\text{seq } X) \rightarrow \text{seq}(\text{seq } X)$
$\forall s, t : \text{seq}(\text{seq } X) \mid \#s = \#t \bullet$
$\forall i : \text{dom } s \bullet$
$(s \simeq t)i = (s_i) \cap (t_i)$

$S\text{Transmit}$
$\Delta \text{Section}$
$m? : M$
$\text{route}' = \text{route}$
$\text{head rec}' = (m?) \cap (\text{head rec})$
$\text{tail rec}' = \text{tail rec}$
$\text{ins}' = \text{ins}$
$\text{sent}' = \text{sent}$

$S\text{Receive}$
$\Delta \text{Section}$
$\text{route}' = \text{route}$
$\text{rec}' = \text{rec}$
$\text{front ins}' = \text{front ins}$
$\text{last ins}' = \text{front}(\text{last ins})$
$\text{front sent}' = \text{front sent}$
$\text{last sent}' = (\text{last}(\text{last ins})) \cap (\text{last sent})$

Consistency

$\exists s : \text{seq } M \bullet \text{in} = s \cap \text{out}$

$\frac{\text{Section}}{\text{head rec} = (\text{in} / \text{ins}) \cap (\text{last sent})^-}$

Base case

$$(\cap / ins) \cap (last sent)$$

$$= (\cap / (ins 1)) \cap (last (sent 1))$$

[#route = #ins = #sent = 1]

$$= (ins 1) \cap (sent 1)$$

$$= rec 1$$

[by definition of $\cap /$]

[from Section]

$$= head rec$$

[by definition of head]

Inductive step

$$head(front rec) = (\cap / (front ins)) \cap (last(front sent))$$

$$(\cap / ins) \cap (last sent)$$

[#ins = #route > 1]

$$= (\cap / ((front ins) \cap (last ins))) \cap (last sent)$$

[#ins = #route > 1]

$$= (\cap / (front ins)) \cap (last ins) \cap (last sent)$$

[by the definition of $\cap /$]

[from Section]

$$= (\cap / (front ins)) \cap (last rec)$$

[by the definition of $\cap /$]

[from Section]

$$= (\cap / (front ins)) \cap (last (tail rec))$$

[#rec = #route > 1]

Retrieve relation

RetrieveExSection

Ext

Section

in = head rec

out = last sent

Transmission

$\forall Ext; Section \mid$
pre Transmit \wedge RetrieveExSection' •

pre Transmit

$\forall Ext; Ext'; Section; Section' \mid$
SectionInit \wedge RetrieveExtSection' •
ExtInit

pre Transmit \wedge RetrieveExSection' •

Transmit

The daemon

$\forall \text{Ext}; \text{Section} \mid$

pre $\exists \text{Ext} \wedge \text{RetrieveExtSection}' \bullet$

pre Daemon

$\forall \text{Ext}; \text{Ext}'; \text{Section}; \text{Section}' \mid$

pre $\exists \text{Ext} \wedge \text{RetrieveExtSection} \wedge$

$\text{Daemon} \wedge \text{RetrieveExtSection}' \bullet$

$\exists \text{Ext}$

Inserting a new section

$\langle a, b, d, e, f \rangle \text{ insert } (2, c) = \langle a, b, c, d, e, f \rangle$

$[X]$

$_insert_ : \text{seq } X \times (\mathbb{N} \times X) \rightarrow \text{seq } X$

$\forall s : \text{seq } X; i : \mathbb{N}; x : X \bullet$

$s \text{ insert } (i, x)$

$=$

$(1 .. i) \triangleleft s \wedge (x) \wedge \text{squash}((1 .. i) \triangleleft s)$

InsertSection

$\Delta \text{Section}$

$s?; \text{new?} : \text{SPC}$

$s? \in \text{ran}(\text{front route})$

$\text{new?} \notin \text{ran route}$

$\exists i : 1 .. (\#\text{route} - 1) \mid$

$i = \text{route}^{\sim} s? \bullet$

$\text{route}' = \text{route insert } (i, \text{new?})$

$\text{rec}' = \text{rec insert } (i, \text{sent } i)$

$\text{ins}' = \text{ins insert } (i, \langle \rangle)$

$\text{sent}' = \text{sent insert } (i, \text{rec } i + 1)$

\sim / ins'

$= \sim / (\text{ins insert } (i + 1, \langle \rangle))$

[property of ins']

$= \sim / ((1 .. i \triangleleft \text{ins}) \wedge (\langle \rangle) \wedge (\text{squash}(1 .. i \triangleleft \text{ins})))$

[property of ins']

$= \sim / (1 .. i) \triangleleft s \wedge \langle \rangle \wedge \sim / \text{squash}((1 .. i) \triangleleft \text{ins})$

[property of $\sim /$]

[properties of \triangleleft and \triangleleft]