Exercises for Chapter 6

Definitions

Exercise 6.1 (Types and abbreviations) Using basic type declarations and abbreviations, introduce expressions to represent

(a) the set of all people

(b) the set of all people living in Oxford

(c) the set of all pairs of integers

(d) the set of all pairs of integers in which each element is strictly greater than zero

(e) the set of all couples, where a couple comprises exactly two people

(f) the set of all parties, where a party is any group of eight or more people

□

Exercise 6.2 (Approximation) If *ideal* is an integer, and *Available* is a set of natural numbers, then *closest* is a number in *Available* such that the absolute value $|\,closest - ideal\,|$ is as small as possible. Assuming that *ideal* and *Available* are constants of your specification, use an axiomatic definition to define *closest*.

(If m is an integer, then the absolute value $|\,m\,|$ is the difference between m and 0. This is equal to m if m is a positive integer, and equal to $-m$ if m is a negative integer.) □

Exercise 6.3 (Strictly positive) We write $\mathbb{N}_1$ to denote the set of all strictly-positive integers. Show how this set may be defined by abbreviation and by axiomatic definition. □

Exercise 6.4 (Primes and composites) In Z, the symbol mod is used to denote the remainder of integer division, and the symbol .. is used to denote number range. For example,

$$9 \bmod 4 = 1 \qquad 2 \,..\, 8 = \{2, 3, 4, 5, 6, 7, 8\}$$

Using these operators,

(a) define the set *Primes*, which consists of all strictly-positive integers that have no divisors apart from 1 and themselves.

(b) define the set *Composites*, which consists of all strictly-positive integers that are not prime.

□

Exercise 6.5 (Not an element of) The symbol $\notin$ is not a primitive object: it can be defined in terms of the symbol for set membership $\in$. Using a generic definition, define $\_ \notin \_$ to be the set of pairs (x, s) such that x is not an element of s.
□

Exercise 6.6 (The empty set revisited) In our use of the empty set symbol, we will invariably omit the actual parameter. By adding suitable parameters constructed from $\mathbb{N}$, show how it is possible for the following expressions to be well-defined:

(a) $\varnothing \in \varnothing$

(b) $\varnothing \subseteq (\varnothing \times \varnothing)$

(c) $(\varnothing \times \{\varnothing\}) \subseteq \varnothing$

□